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Venue Hong Kong Convention and Exhibition Centre

Overlap Region Extraction of Two NURBS Surfaces

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Sponsored by acm

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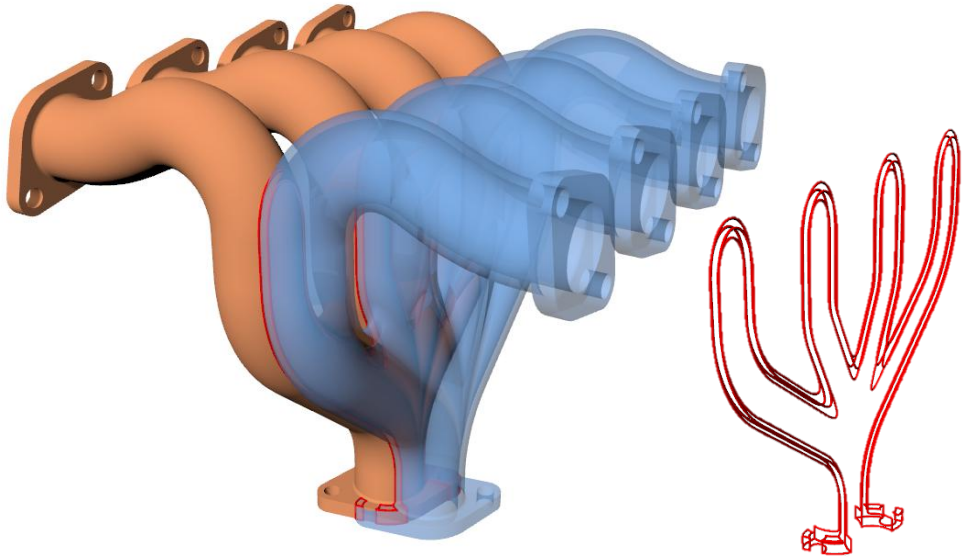
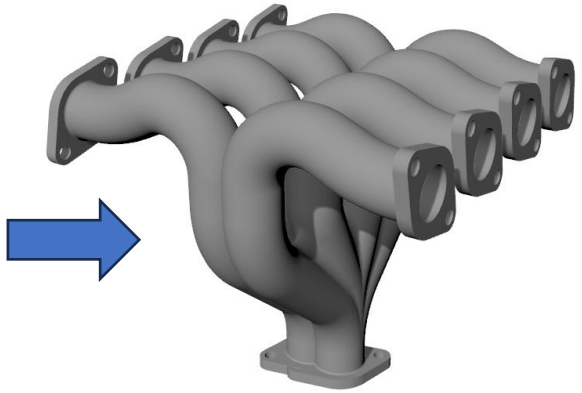
INTRODUCTION

B-Rep Booleans in regular cases

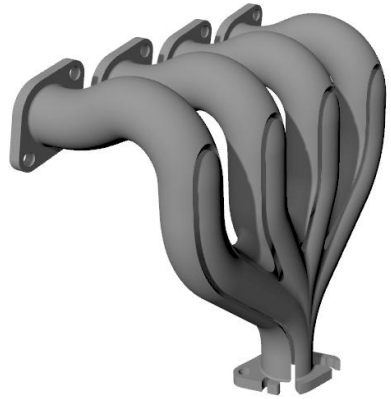


Models

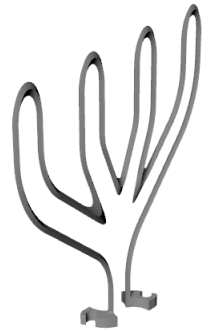
Intersection



$A \cup B$

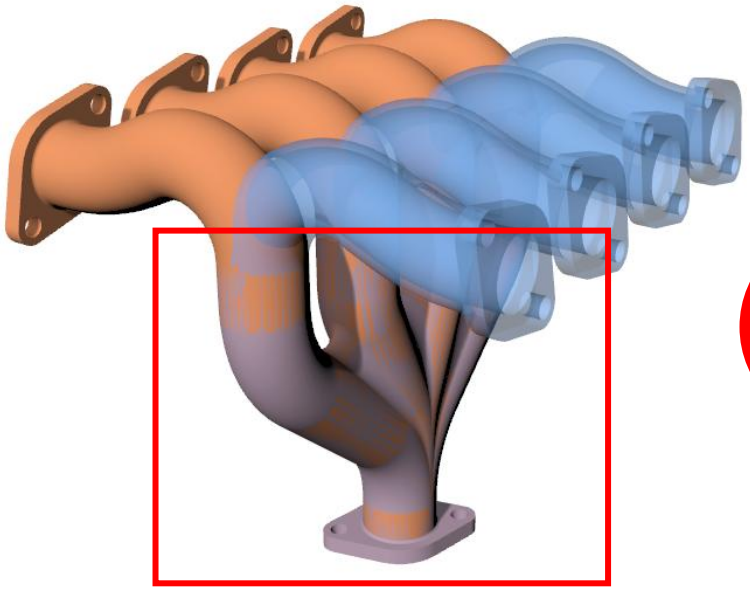


$A \setminus B$



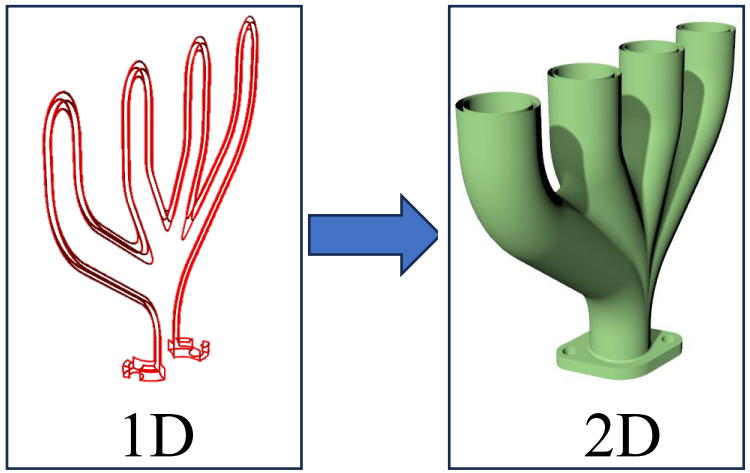
$A \cap B$

B-Rep Booleans with **overlap** surfaces

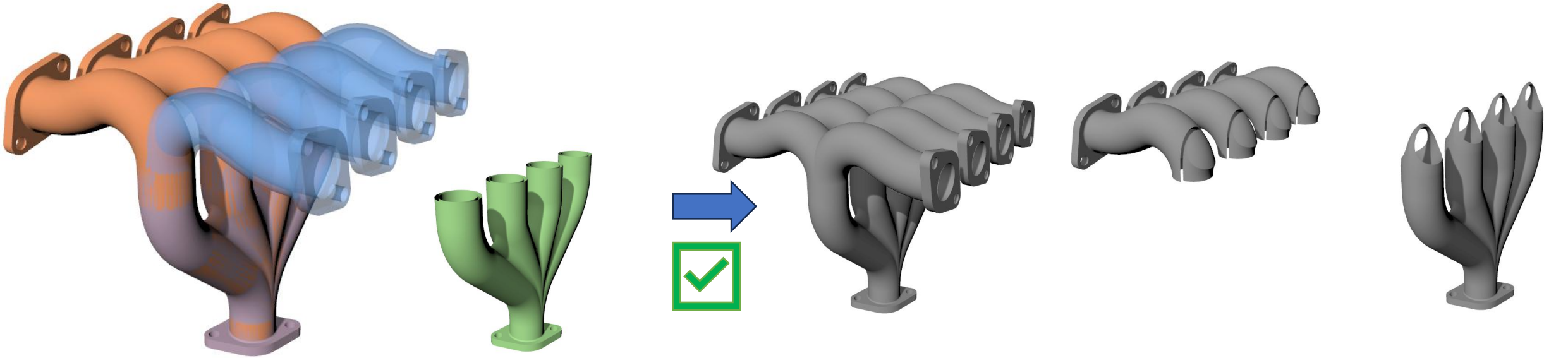


Models

Intersection



B-Rep Booleans with **overlap** detection

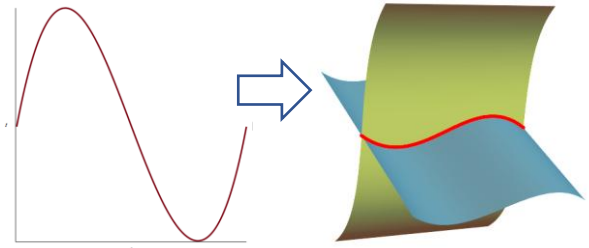


Models | Overlap | A ∪ B | A\B | A ∩ B

RELATED WORK

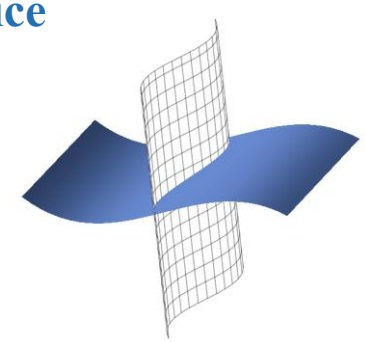
Surface Intersection

Algebraic



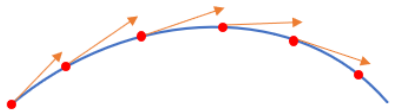
[Manocha and Canny 1991]

Lattice



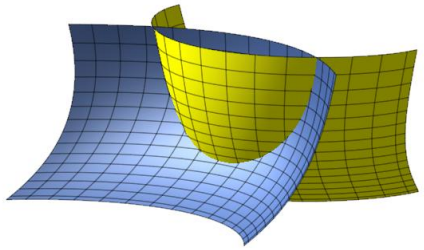
[Rossignac and Requicha, 1987]

Marching



[Barnhill and Kersey 1990]

Subdivision

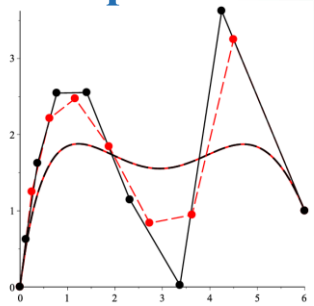


[Figueiredo, 1996]

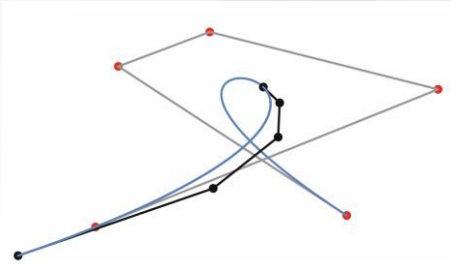
Inapplicable to overlap surfaces, limited speed

Overlap Region Detection

Curve overlap detection

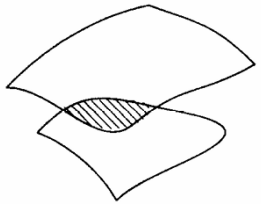


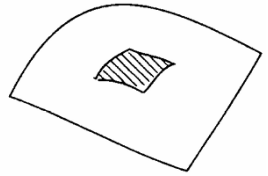
[Chen et al. 2016]



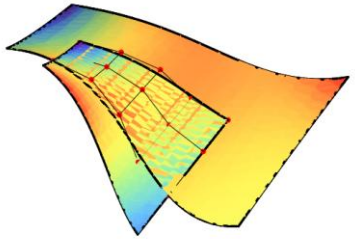
[Vlachkova 2017]

Surface overlap detection





[Hu et al. 1997]

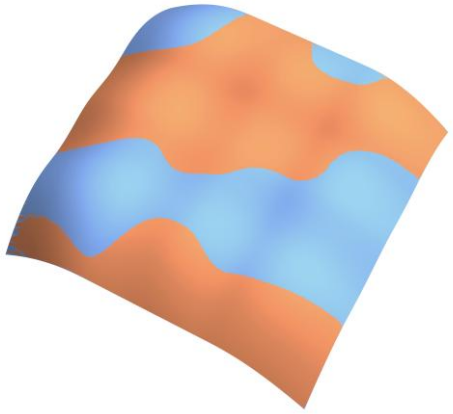


[Vlachkova 2023]

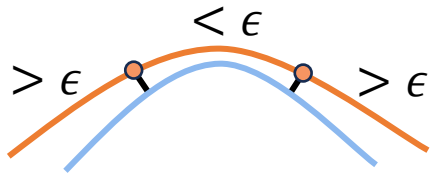
Algebraic dependence, lack of tolerance overlap

Goal: Tolerance-based Overlap Extraction

Input



(1) NURBS Surfaces

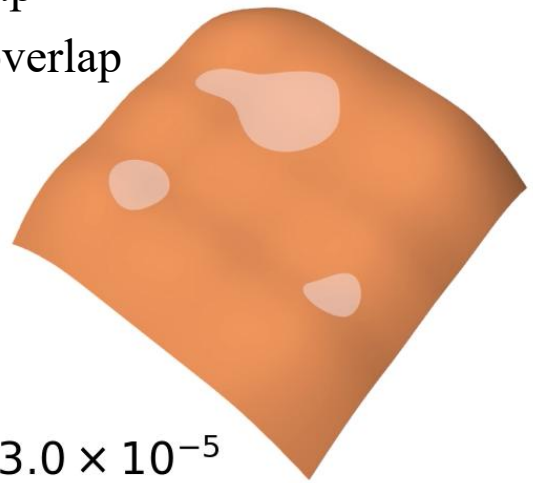


(2) Tolerance ϵ

Output

overlap

non-overlap



$\varepsilon = 3.0 \times 10^{-5}$

- Overlap Region within Tolerance ϵ
- Based on Floating-point Representation

FORMULATION

Problem Formulation

Point-to-Point Distance:

$$d_0(u, v, s, t) = \|\mathbf{r}_1(u, v) - \mathbf{r}_2(s, t)\|_2$$

Point-to-Surface Distance:

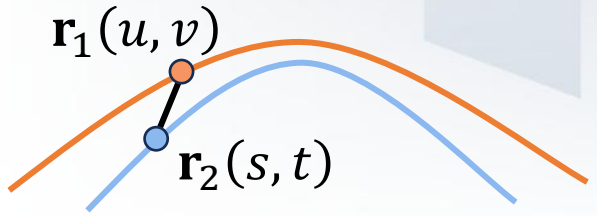
$$d_2(u, v) = \min_{(s,t)\in\mathcal{D}_2} d_0(u, v, s, t)$$

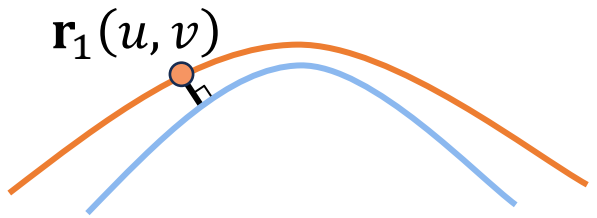
Overlap Region within Given Tolerance ϵ:

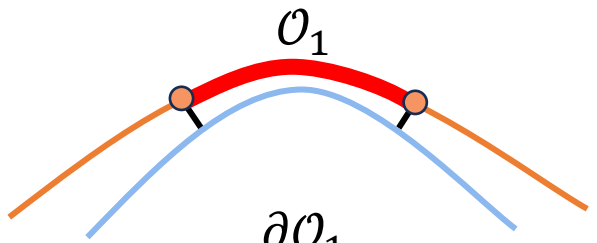
$$\mathcal{O}_1 := \{(u, v) \in \mathcal{D}_1 \mid d_2(u, v) \leq \epsilon\}$$

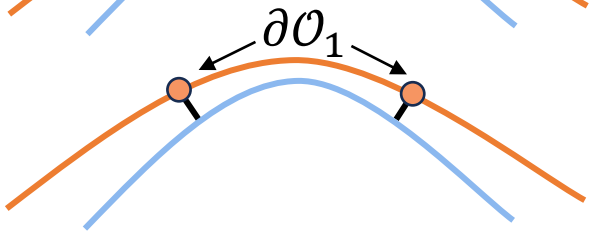
Overlap Region Boundary:

$$\partial\mathcal{O}_1 := \{(u, v) \in \mathcal{D}_1 \mid d_2(u, v) = \epsilon\}$$









Bilevel Optimization Framework

$$\partial\mathcal{O}_1 := \{(u,v) \in \mathcal{D}_1 | \min_{(s,t)\in\mathcal{D}_2} d_0(u,v,s,t) = \epsilon\}$$

Bilevel Optimization

Upper Level		Lower Level
$\min_{(u,v)\in\mathcal{D}_1} f^U(u,v,s,t)$	subject to	$(s,t) \in \arg\min_{(s',t')\in\mathcal{D}_2} f^L(u,v,s',t')$

f^U: $d_2(u,v) = \epsilon$

f^L: Point-to-Surface Distance d_2

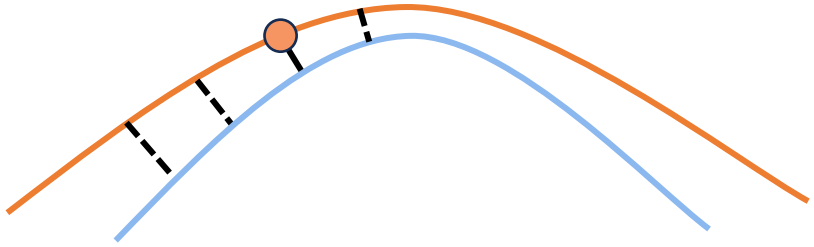
Bilevel Optimization Framework

$$\partial\mathcal{O}_1 := \{(u,v) \in \mathcal{D}_1 | \min_{(s,t)\in\mathcal{D}_2} d_0(u,v,s,t) = \epsilon\}$$

Bilevel Optimization

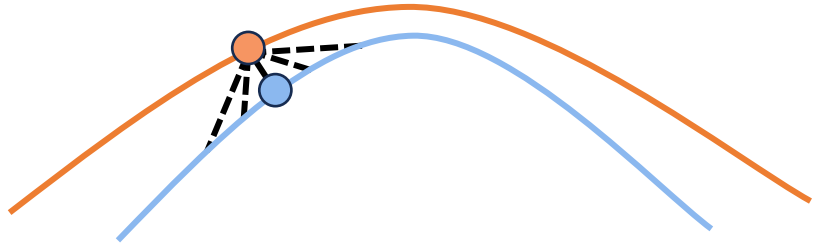
Upper Level

Find $(u,v) \in \mathcal{D}_1$, s.t. $d_2(u,v) = \epsilon$



Lower Level

For each $(u,v) \in \mathcal{D}_1$, Find closest $(s,t) \in \mathcal{D}_2$



ALGORITHM

Algorithm Pipeline

Input:

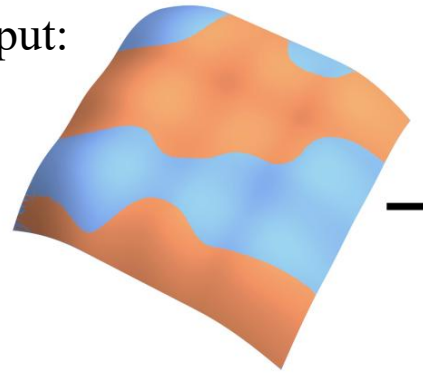
Output:

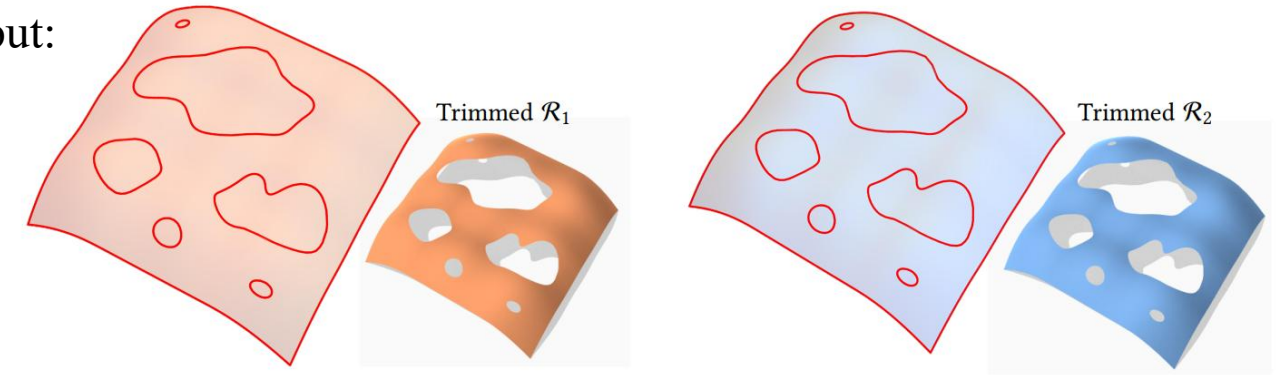
Result in $\mathcal{R}_1$

Trimmed $\mathcal{R}_1$

Result in $\mathcal{R}_2$

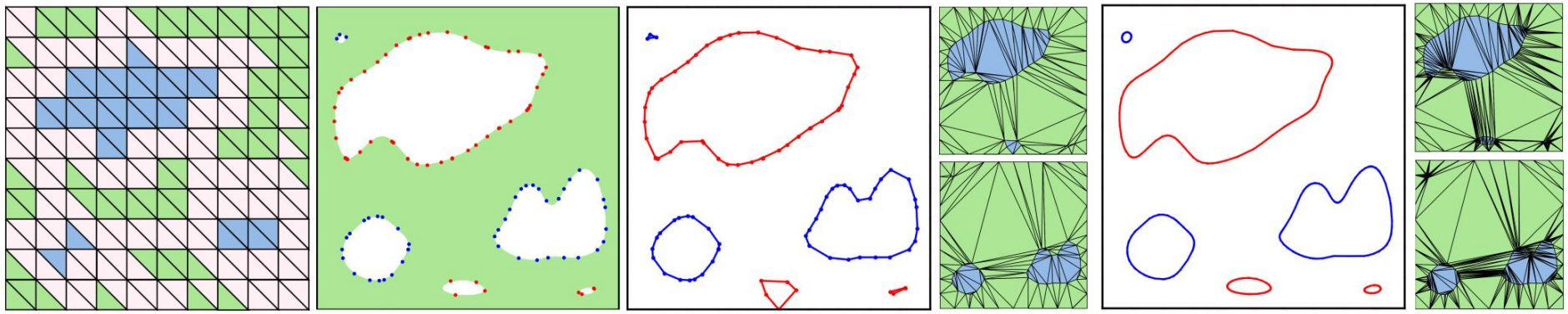
Trimmed $\mathcal{R}_2$





Cartesian Space $\mathbb{R}^3$

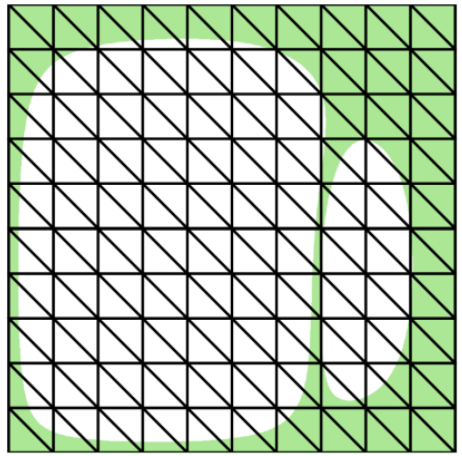
Parametric Domain $\mathcal{D}_1$

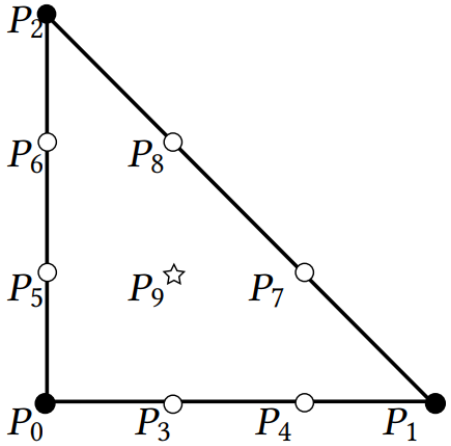


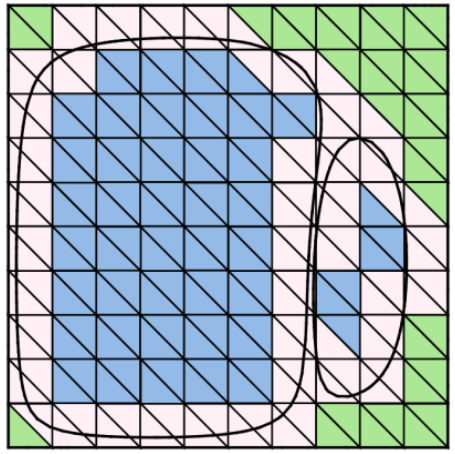
Boundary Region Estimation	Boundary Point Calculation	Boundary Topology Connection	Boundary Edge Refinement

Boundary Region Estimation	Boundary Point Calculation	Boundary Topology Connection	Boundary Edge Refinement

Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**

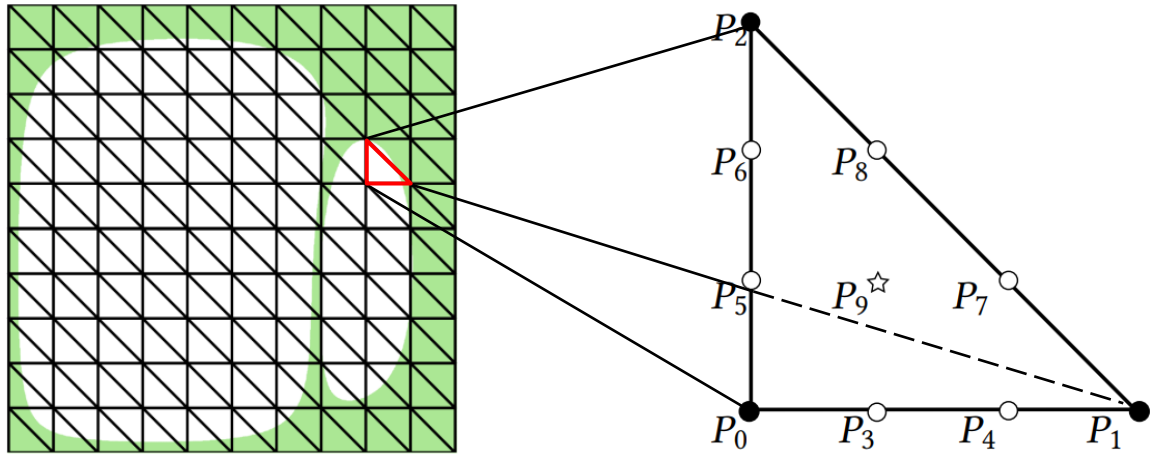


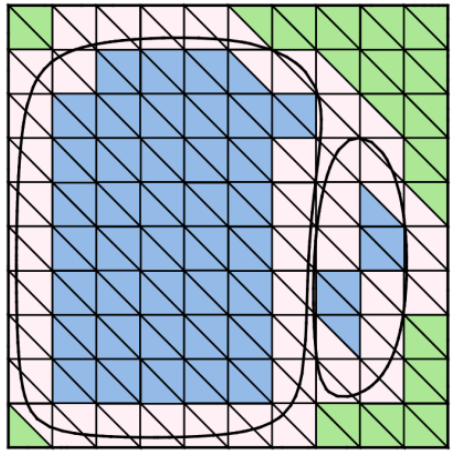




Triangulation

Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**

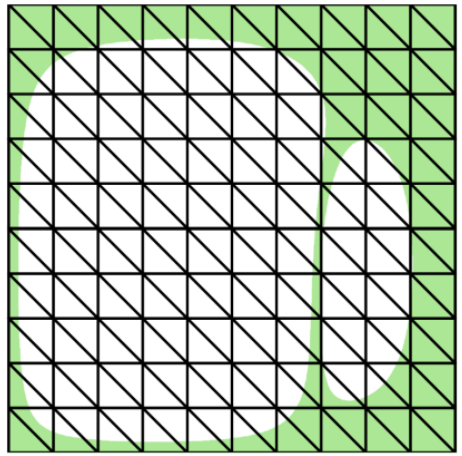


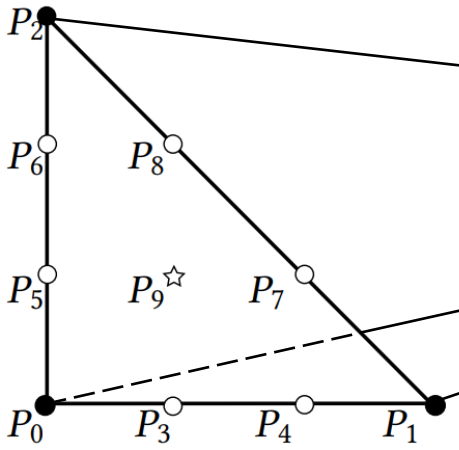


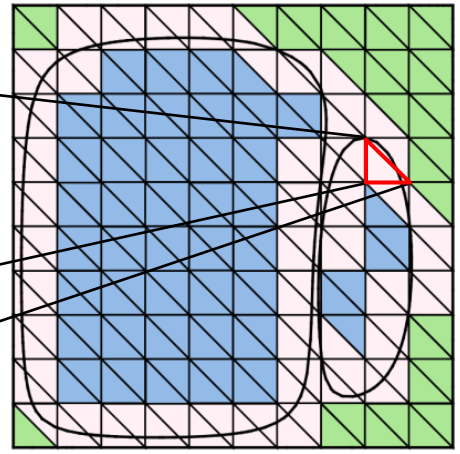
Triangulation → **Estimation**

cubic Bézier simplex interpolation

Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**







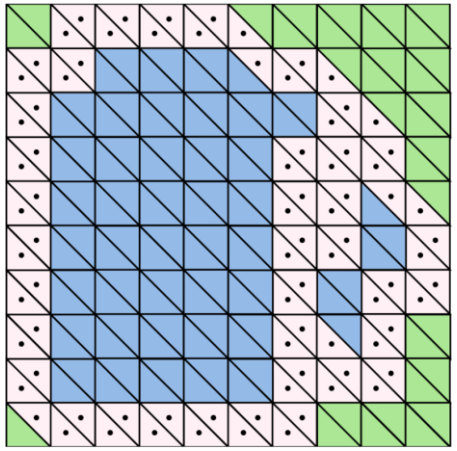
Triangulation → **Estimation** → **Classification**

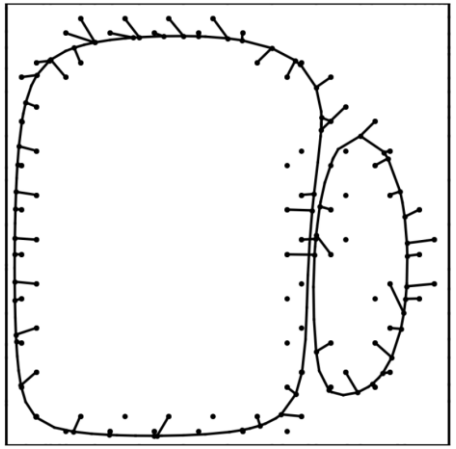
inside $\mathcal{O}_1$

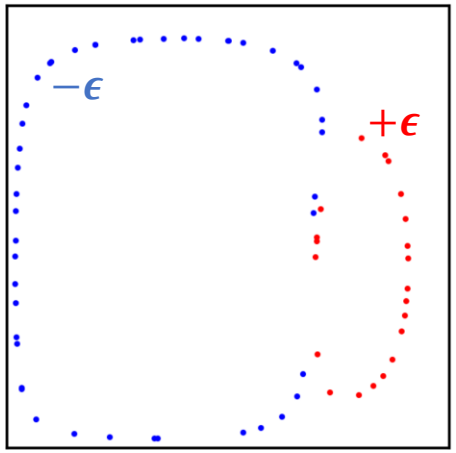
outside $\mathcal{O}_1$

intersect $\partial\mathcal{O}_1$

Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**

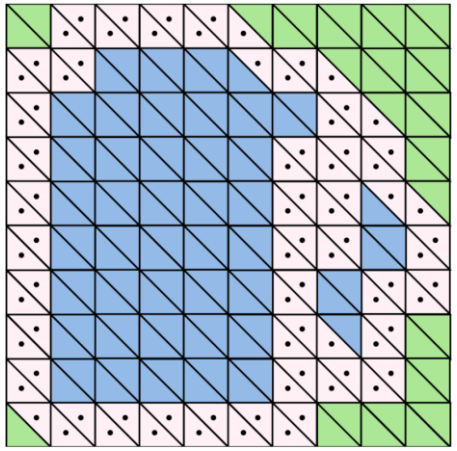


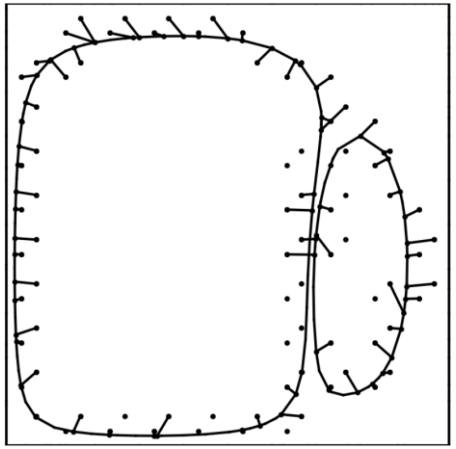


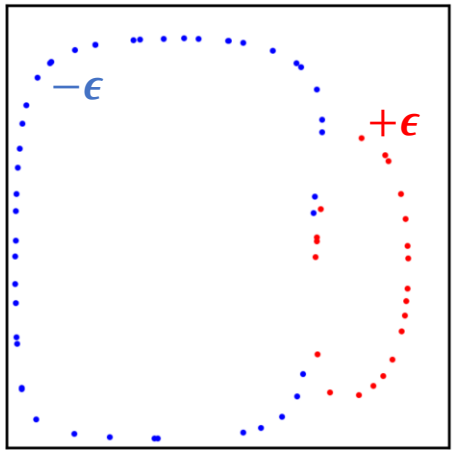


Initial Guess

Boundary Region Estimation → **Boundary Point Calculation** → Boundary Topology Connection → Boundary Edge Refinement

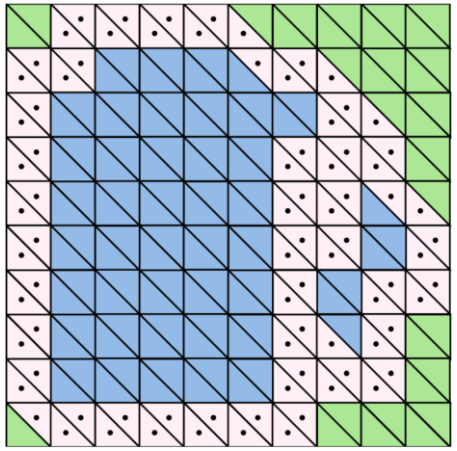


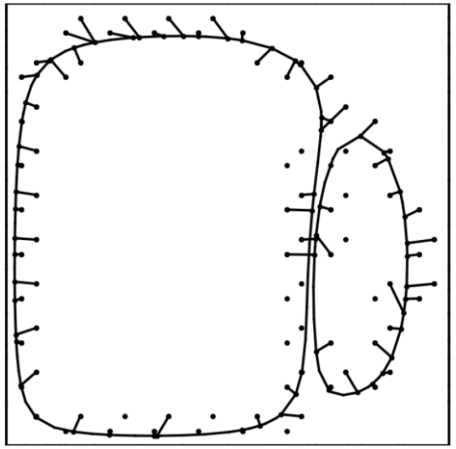


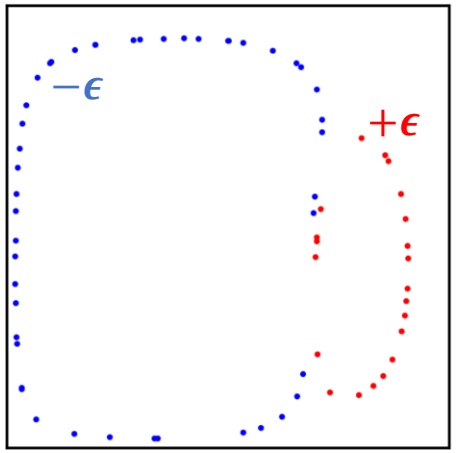


Initial Guess → Optimization

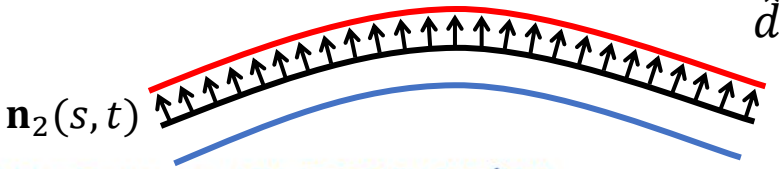
Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**







Initial Guess → **Optimization** → **Classification**

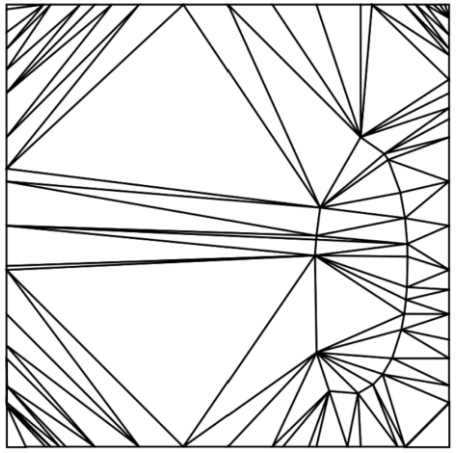


$$\hat{d}(u,v) \coloneqq \big(\mathbf{r}_1(u,v) - \mathbf{r}_2(s,t)\big) \cdot \mathbf{n}_2(s,t)$$

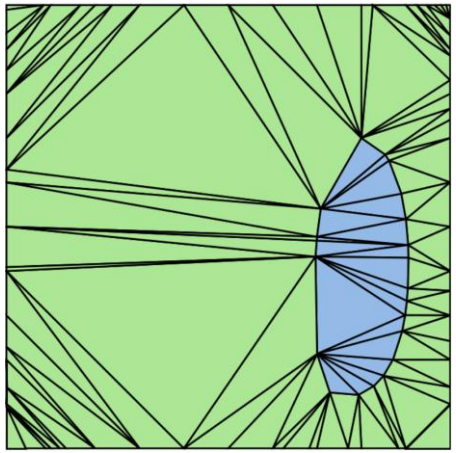
$+\epsilon$ branch: $\hat{d}(u,v) \geq 0$

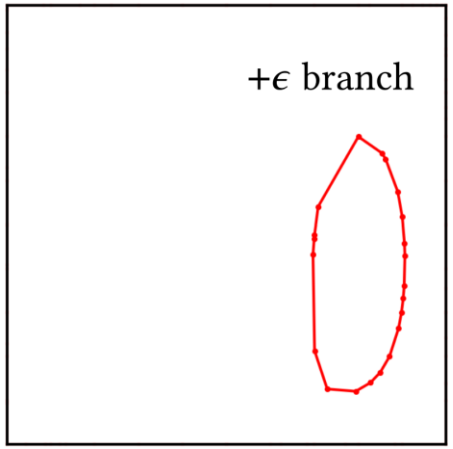
$-\epsilon$ branch: $\hat{d}(u,v) < 0$

Boundary Region Estimation → Boundary Point Calculation → **Boundary Topology Connection** → Boundary Edge Refinement

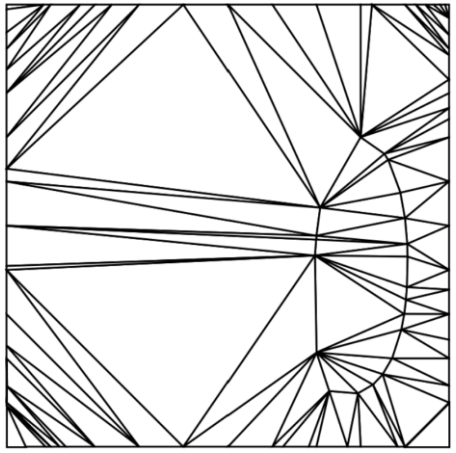


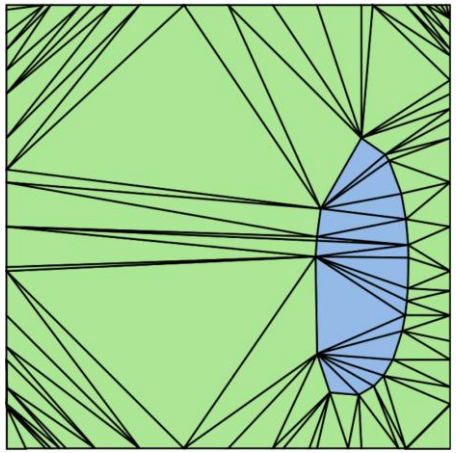
Delaunay Triangulation

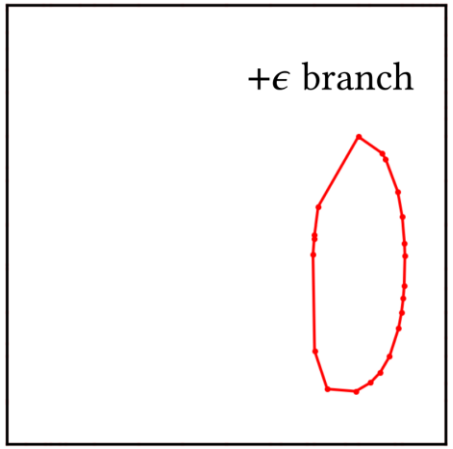




Boundary Region Estimation → Boundary Point Calculation → **Boundary Topology Connection** → Boundary Edge Refinement



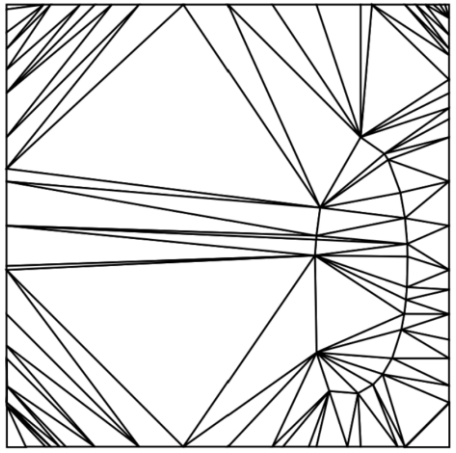


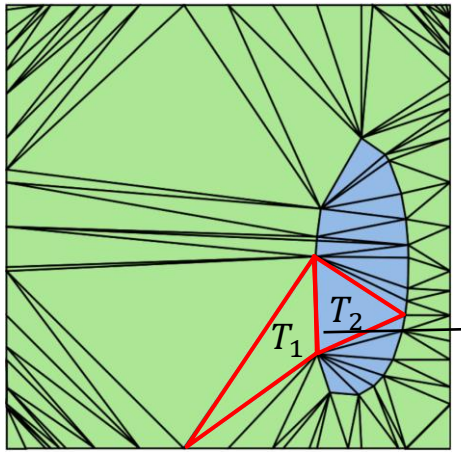


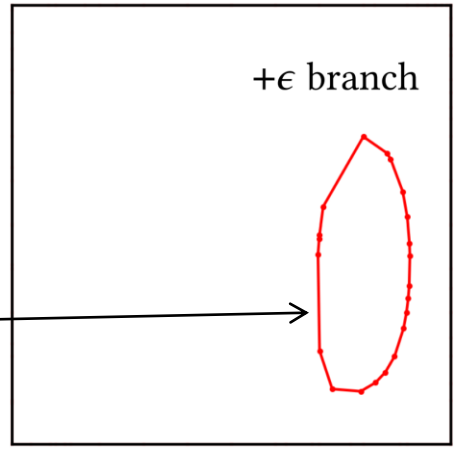
Delaunay Triangulation → **Classification**

$$1_{\mathcal{O}_+}(T_i) = \begin{cases} 1 & \text{if } T_i \in \mathcal{O}_+ \\ 0 & \text{if } T_i \notin \mathcal{O}_+ \end{cases}$$

Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**





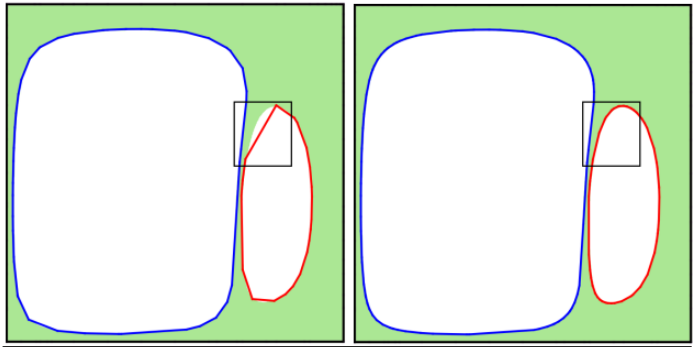


Delaunay Triangulation → **Classification** → **Boundary Edge Identification**

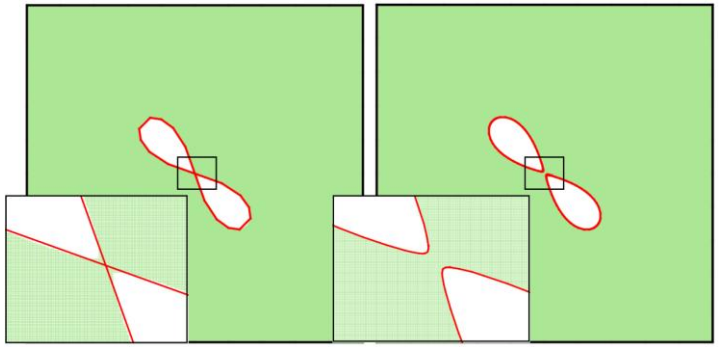
$$1_{\mathcal{O}_+}(T_i) = \begin{cases} 1 & \text{if } T_i \in \mathcal{O}_+ \\ 0 & \text{if } T_i \notin \mathcal{O}_+ \end{cases}$$

$$1_{\mathcal{O}_+}(T_1) \neq 1_{\mathcal{O}_+}(T_2)$$

Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**

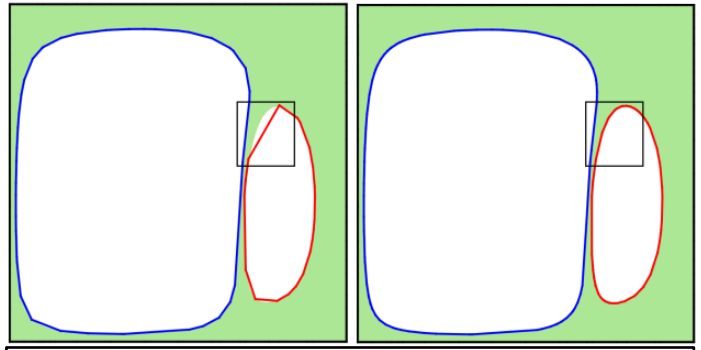


Significant Discrepancies

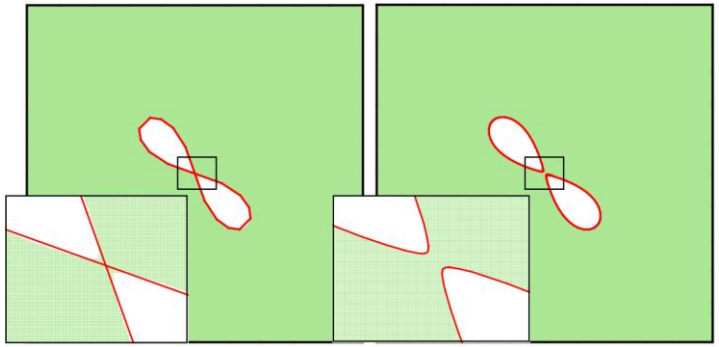


Incorrect Topology

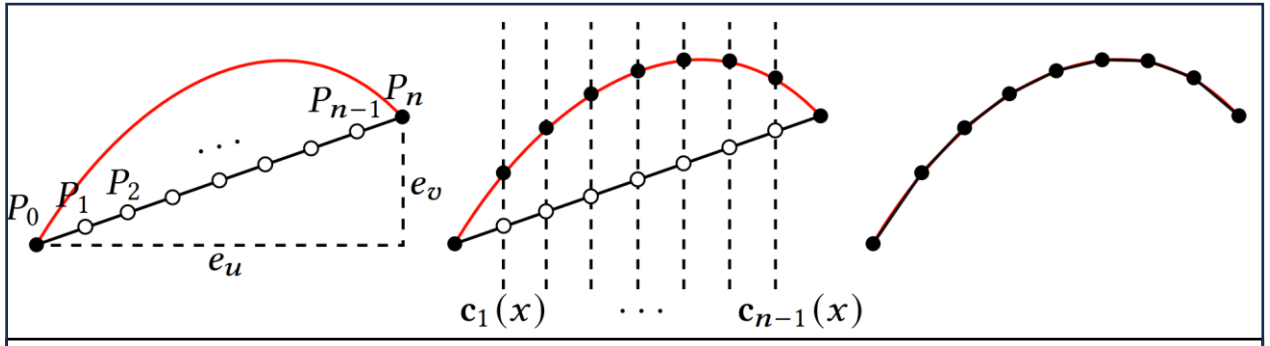
Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**



Significant Discrepancies

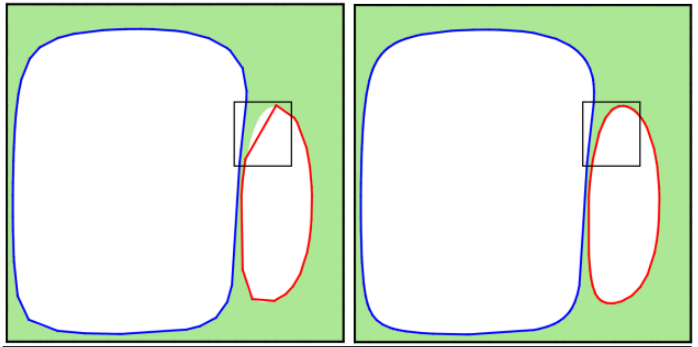


Incorrect Topology

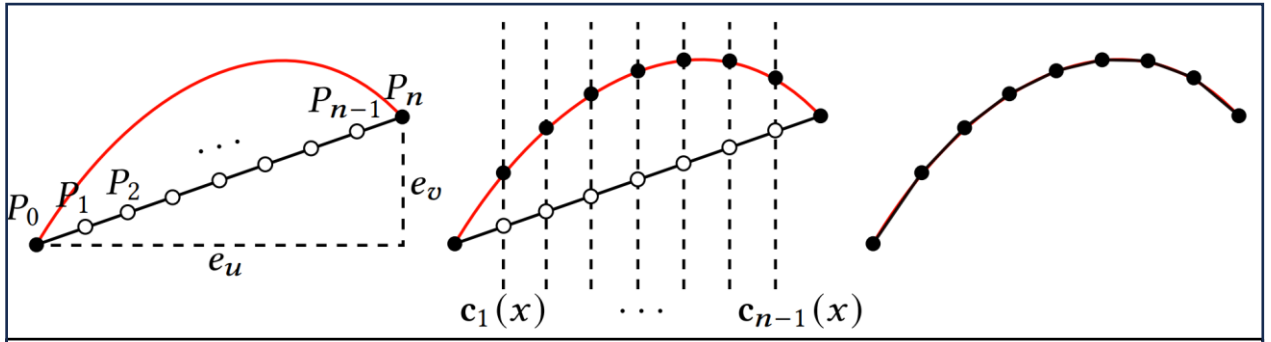


Length-based Refinement

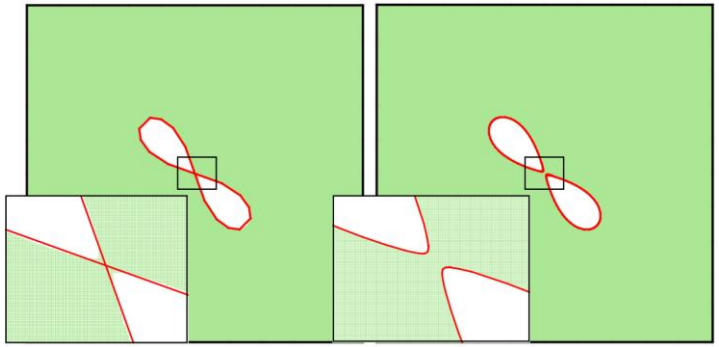
Boundary Region Estimation → **Boundary Point Calculation** → **Boundary Topology Connection** → **Boundary Edge Refinement**



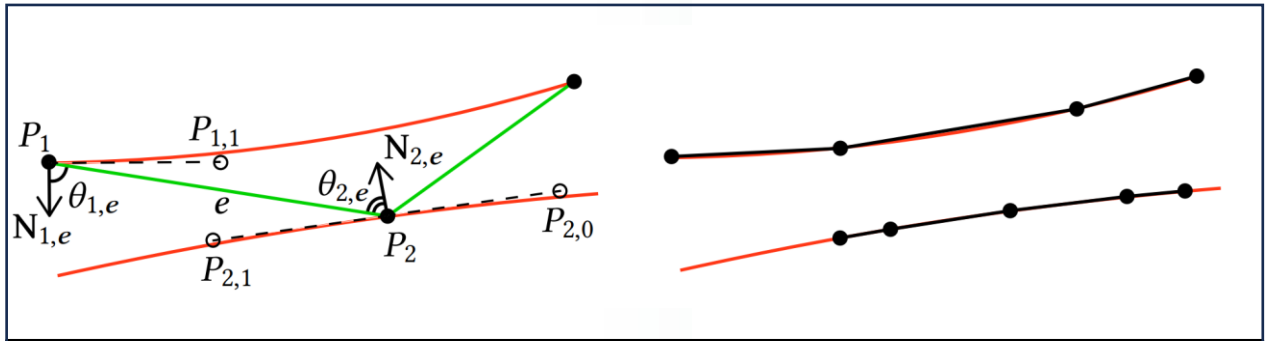
Significant Discrepancies



Length-based Refinement



Incorrect Topology



Normal-based Refinement

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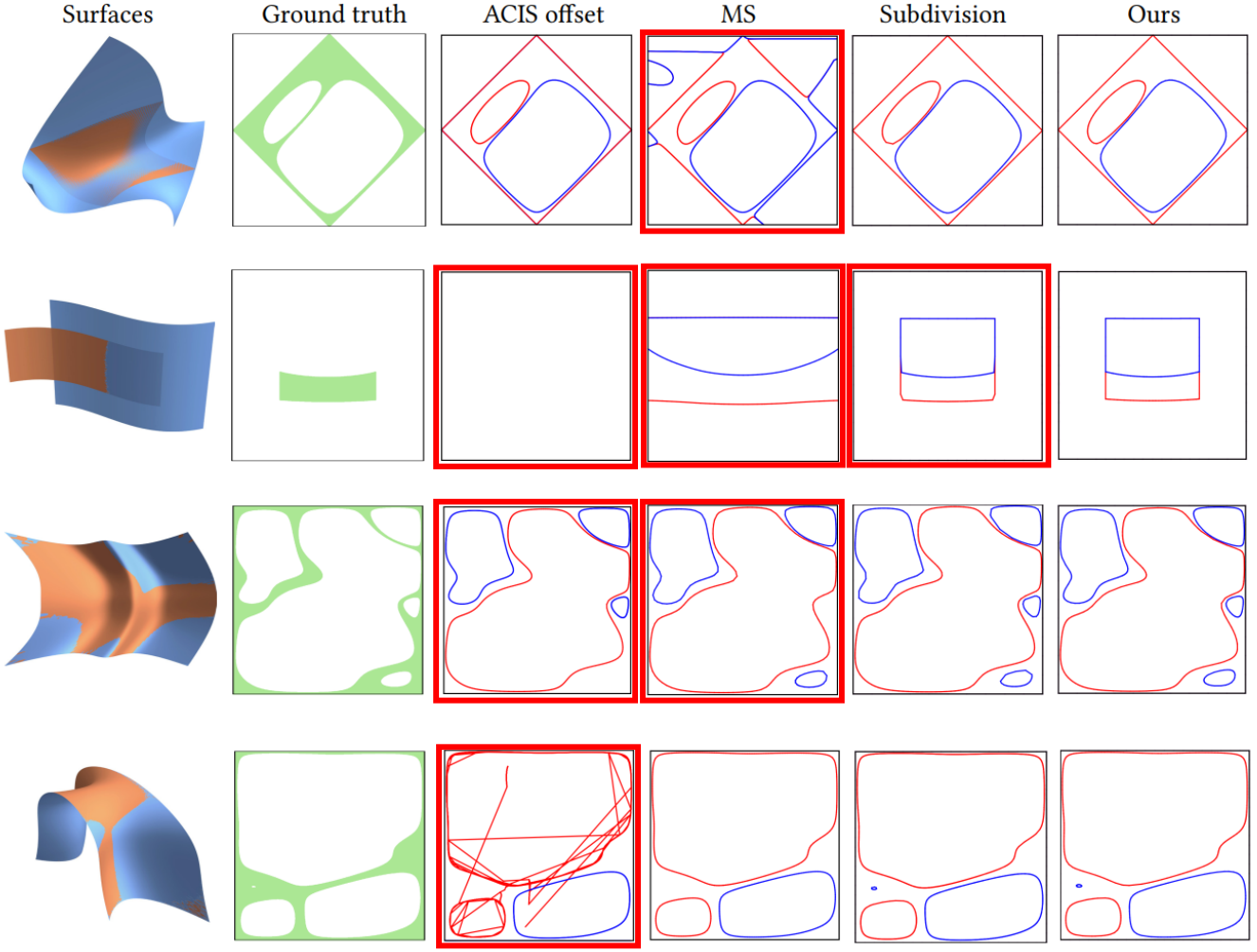
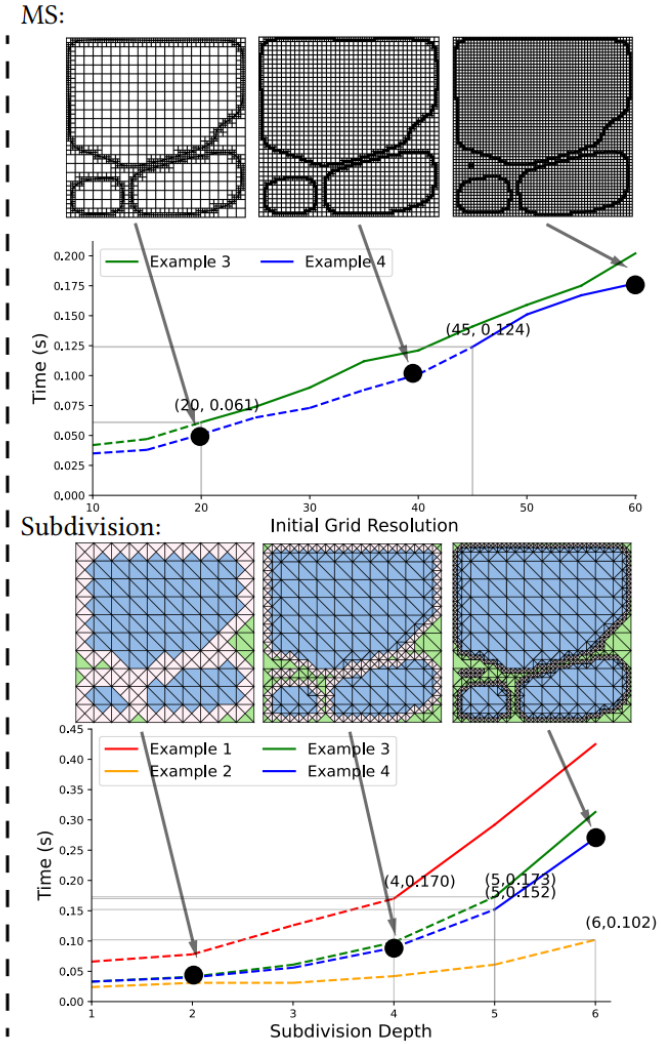
RESULTS

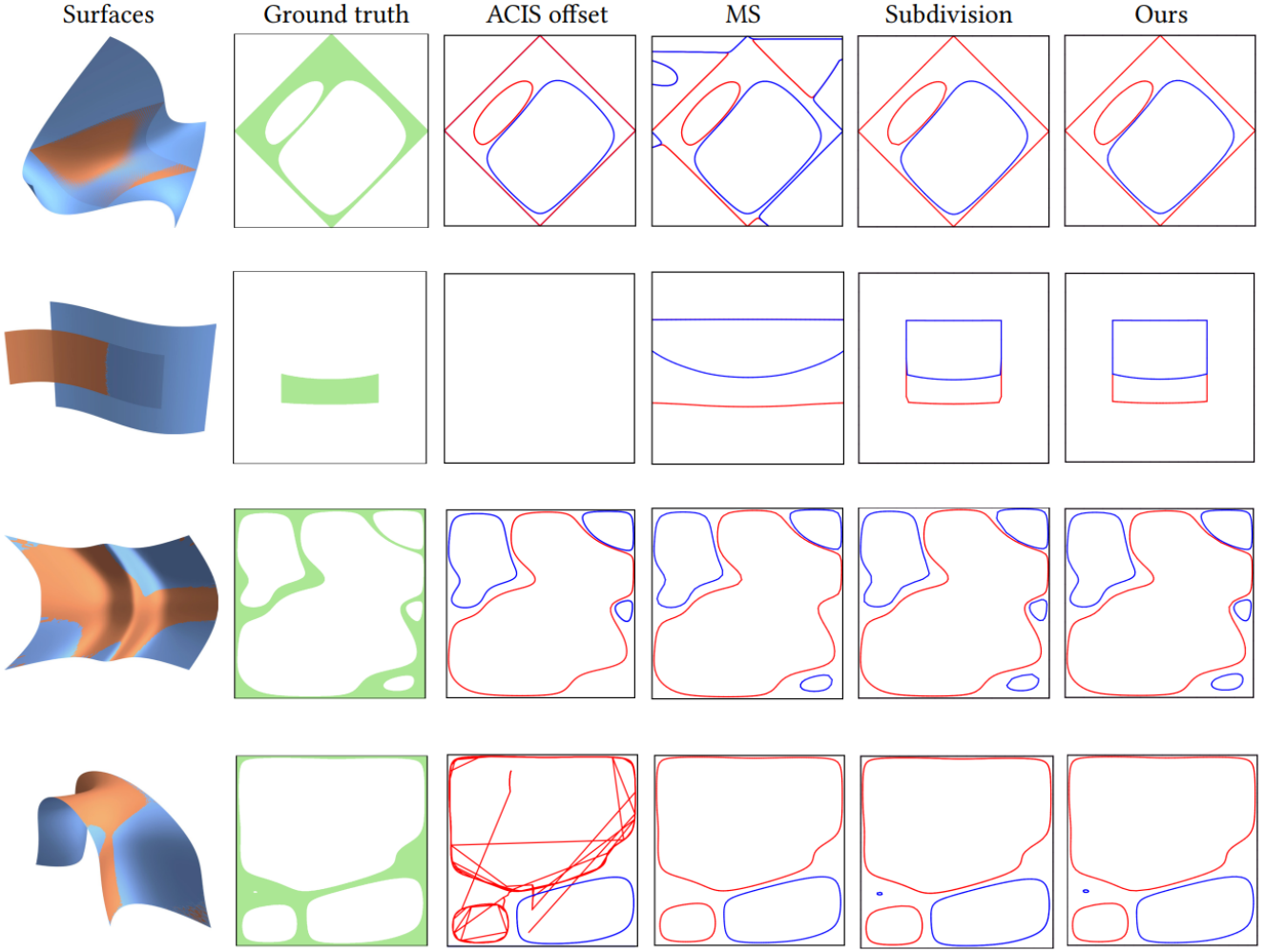
Performance Comparisons



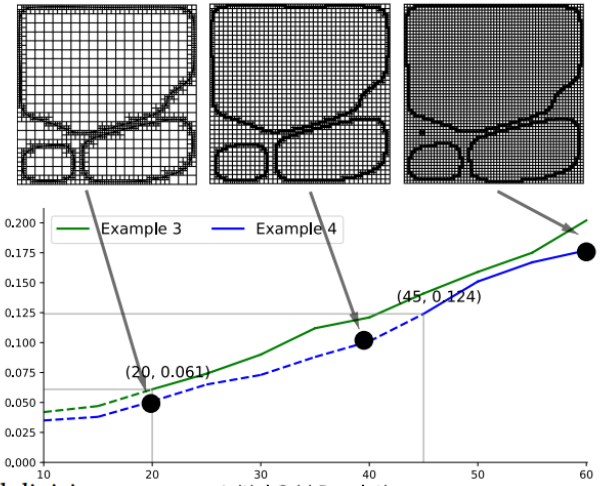


Performance Comparisons

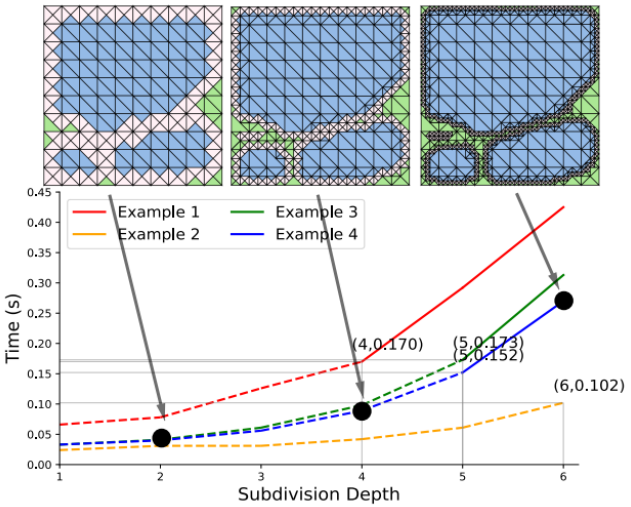
Example	Time and Correctness			
	ACIS	MS	Subdivision	Ours
1	2.846 (T)	0.224 (F)	0.292 (T)	**0.145** (T)
2	0.005 (F)	0.051 (F)	0.061 (F)	**0.056** (T)
3	0.090 (F)	0.042 (F)	0.173 (T)	**0.077** (T)
4	0.613 (F)	0.035 (F)	0.152 (T)	**0.087** (T)



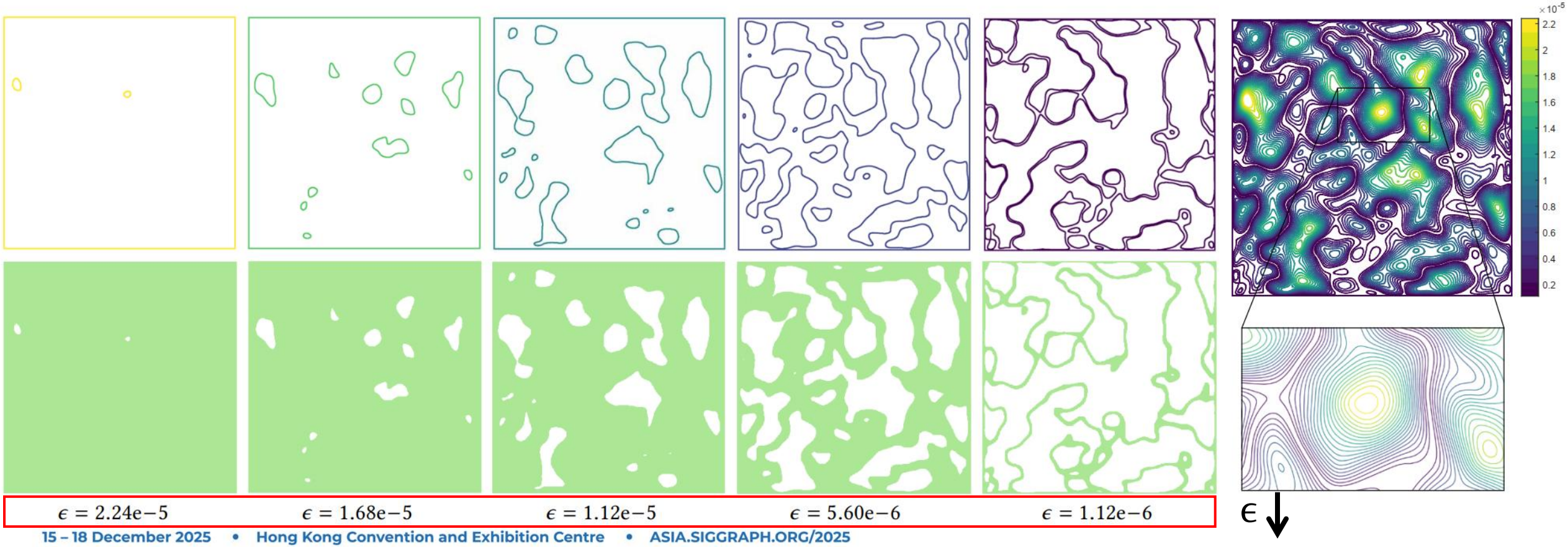
MS:



Subdivision:



Different Tolerances



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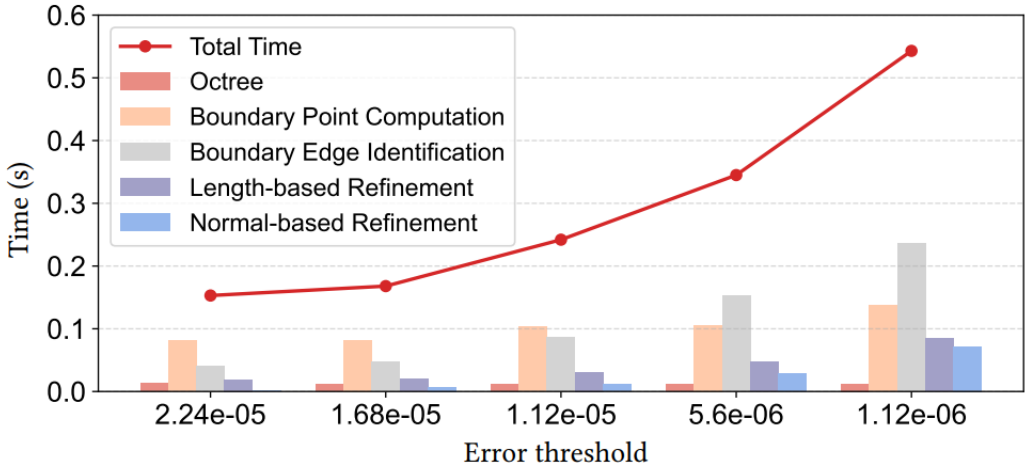
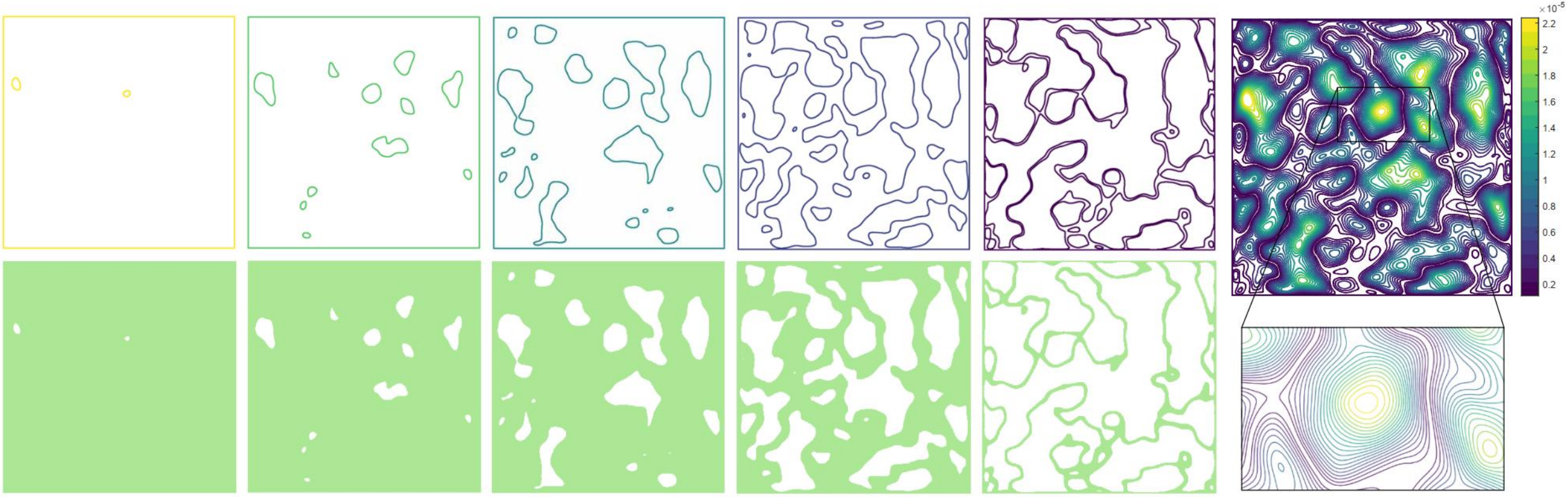
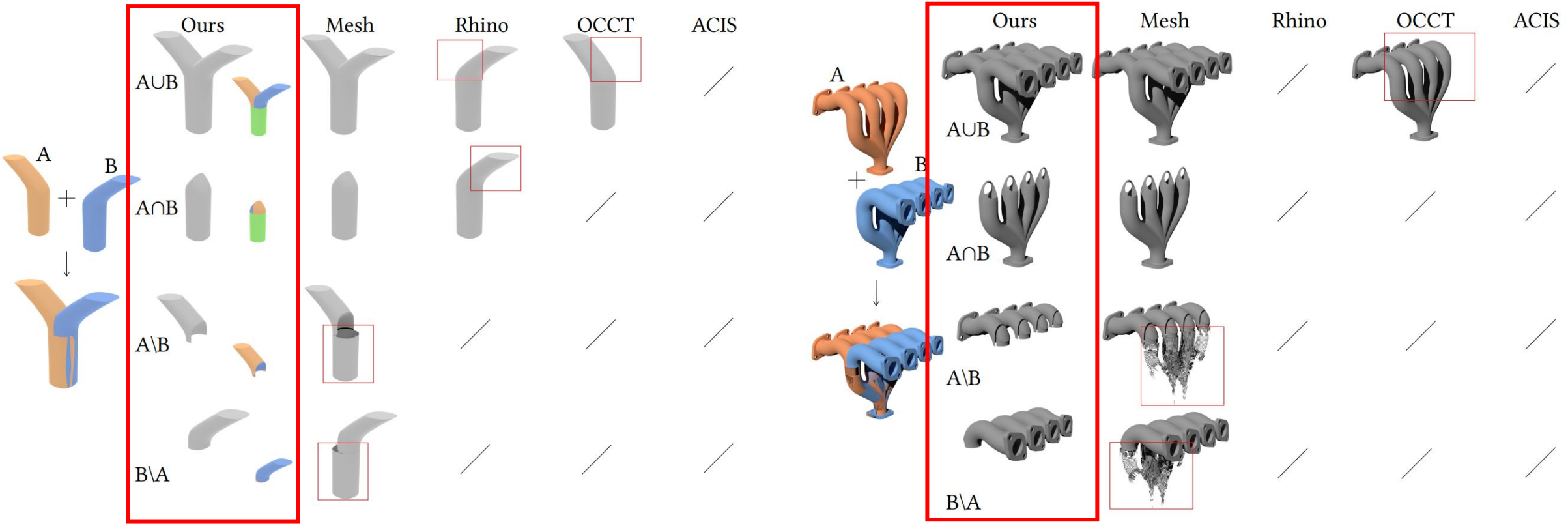
Different Tolerances





$\epsilon = 2.24e{-}5$ $\epsilon = 1.68e{-}5$ $\epsilon = 1.12e{-}5$ $\epsilon = 5.60e{-}6$ $\epsilon = 1.12e{-}6$

Application



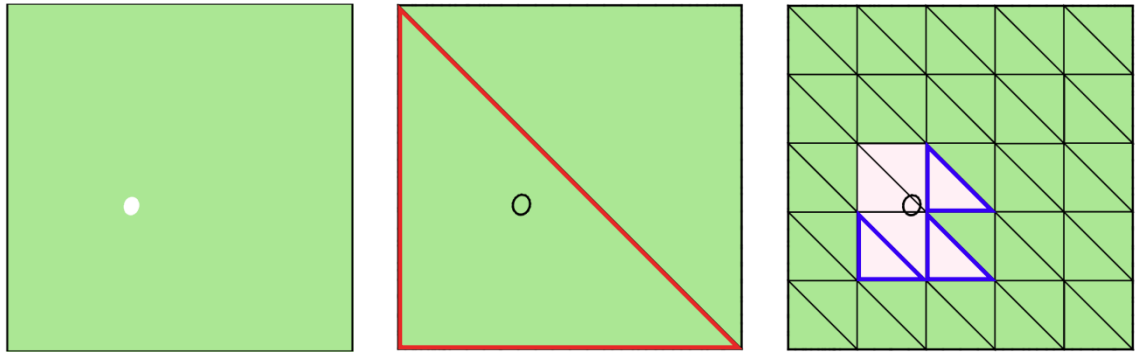
CONCLUSION

Conclusion

- Introduce **tolerance** in NURBS Surface overlap region
- Propose a **bilevel optimization framework** for overlap region calculation
- Present an **efficient and robust algorithm** for overlap region calculation

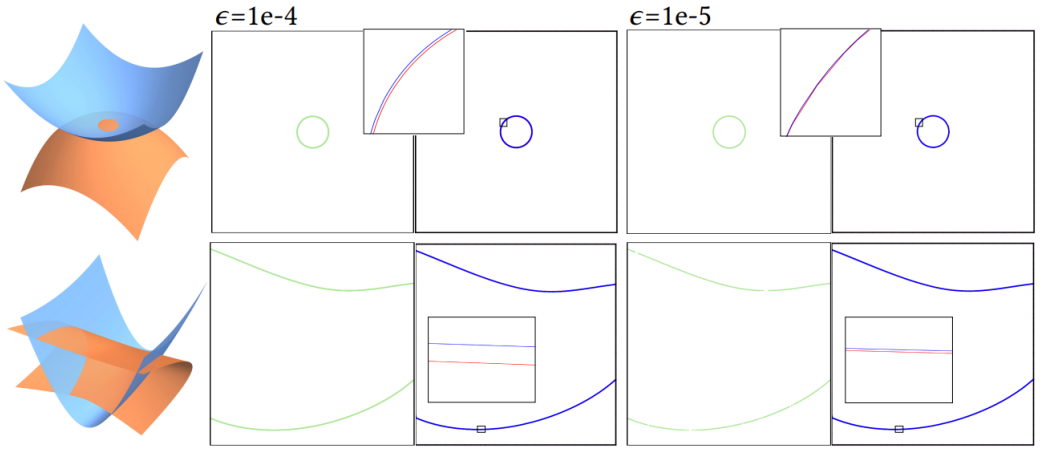
Limitation

(a) Misclassification



(a) Ground truth (b) Sparse triangulation (c) Dense triangulation

(b) Handling transversal intersection surfaces



THANK YOU!

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