SIGGRAPH ASIA 2023 SYDNEY

TOPOLOGY GUARANTEED B-SPLINE SURFACE/SURFACE INTERSECTION

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Connecting STORIES

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EXHIBITION 13 - 15 December 2023

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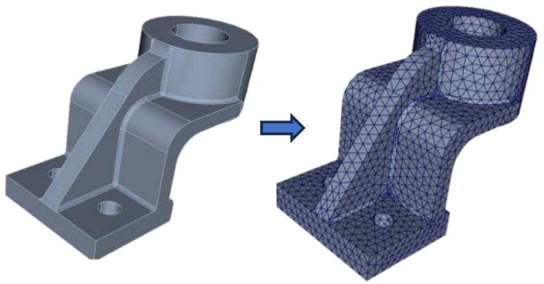
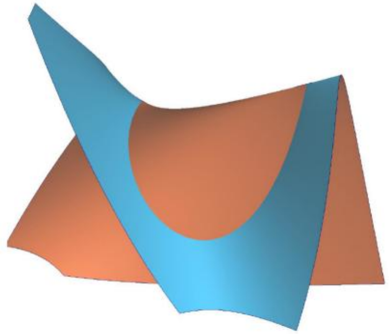
INTRODUCTION

Background



Mesh generation

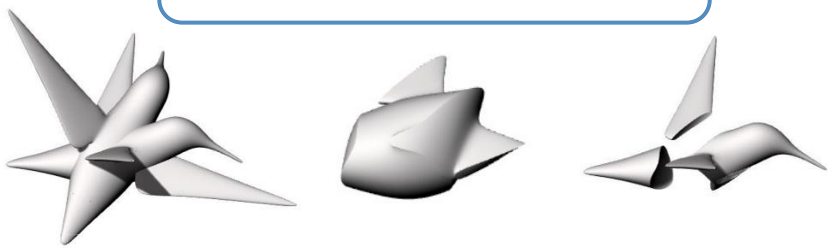
Rendering

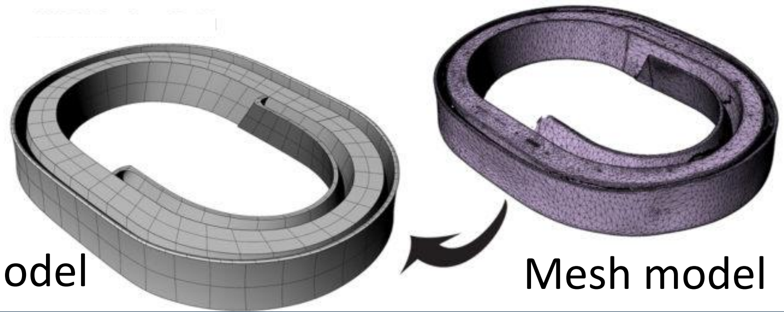


Surface Intersection

Boolean operations

Reverse engineering





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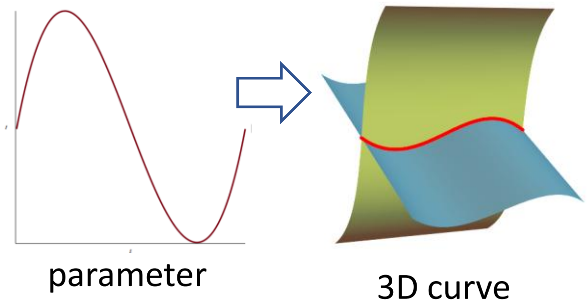
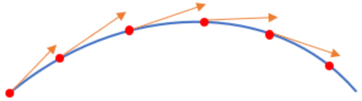
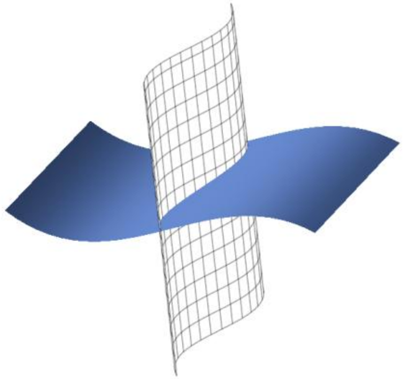
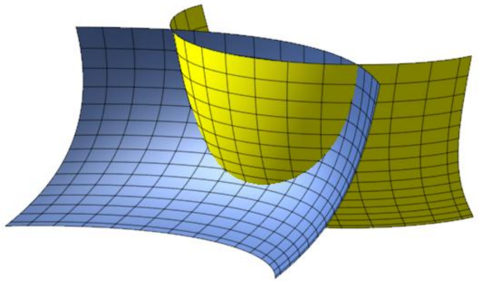
Research Since 1980

Algebraic	Marching	Lattice	Subdivision









[Manocha and Canny 1991] [Barnhill and Kersey 1990] [Rossignac and Requicha, 1987] [Figueiredo, 1996]

Challenges

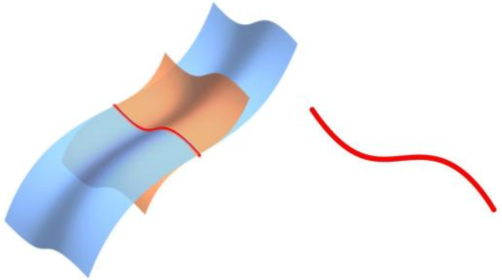
Limitations of classical methods

Algebraic → implicitization, parametric domain

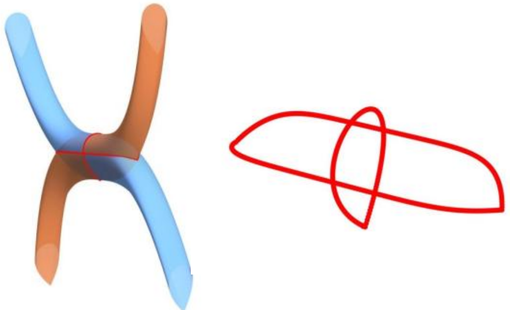
Marching → starting points, tracing directions

Lattice, Subdivision → tangent curves, isolated points

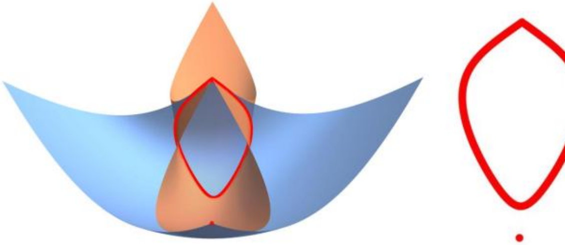
The topology we can handle



high order tangency



multiple branches



isolated point

PRELIMINARIES

Parametric Surface

Parametric representation of the B-Spline surface

$$\mathbf{P}(u,v) = \left(\frac{a(u,v)}{d(u,v)}, \frac{b(u,v)}{d(u,v)}, \frac{c(u,v)}{d(u,v)}\right) \in \mathbb{R}(u,v)^3$$

Homogeneous form

$$\mathbf{P}(u,v) = \big(a(u,v), b(u,v), c(u,v), d(u,v)\big) \in \mathbb{RP}[u,v]^3$$

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Moving plane

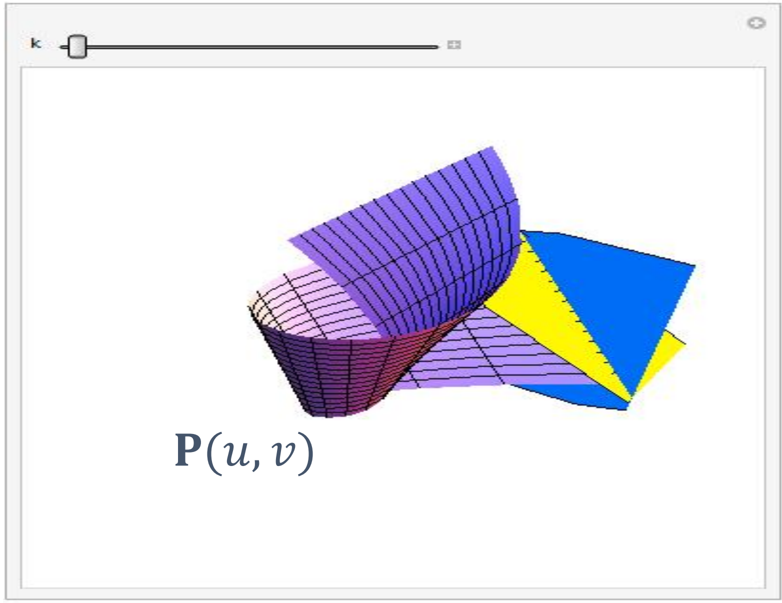
$$\boxed{L(\boldsymbol{X}; u, v) := A(u,v)x + B(u,v)y + C(u,v)z + D(u,v)w = 0}$$

Follow the parametric surface $\mathbf{P}(u, v)$

$$\boxed{L(\mathbf{P}(u,v); u, v) \equiv 0}$$

Example

$$f(\boldsymbol{X}; u, v) \coloneqq d(u,v)x - a(u,v)w$$



“Implicitization using moving curves and surfaces.” Sederberg & Chen, 1995

Dixon Matrix

Dixon matrix

$$\frac{1}{(u-\alpha)(v-\beta)}\begin{vmatrix} f(u,v) & g(u,v) & h(u,v) \\ f(u,\beta) & g(u,\beta) & h(u,\beta) \\ f(\alpha,\beta) & g(\alpha,\beta) & h(\alpha,\beta) \end{vmatrix} = \left(1,\alpha,\beta,\cdots,\alpha^{m-1}\beta^{2n-1}\right)\boldsymbol{D}(\boldsymbol{f},\boldsymbol{g},\boldsymbol{h})\begin{pmatrix} 1 \\ u \\ v \\ \vdots \\ u^{2m-1}v^{n-1} \end{pmatrix}$$

Dixon resultant

$$\boxed{\operatorname{res}(f,g,h) \coloneqq \det\big(D(f,g,h)\big)} \Rightarrow \text{Elimination}$$

"Elimination and resultants. 1. elimination and bivariate resultants." Chionh & Goldman, 1995

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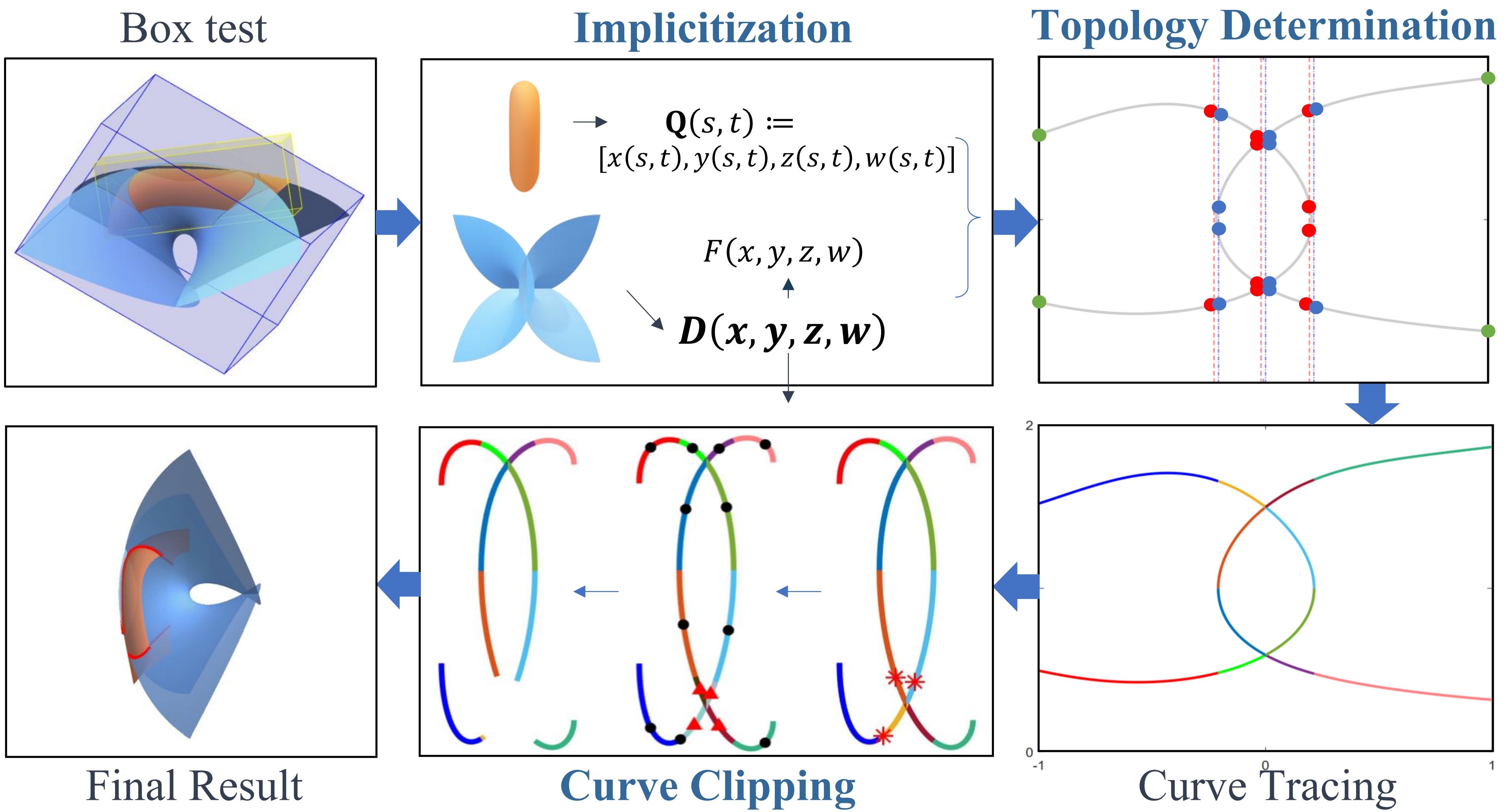
ALGORITHM

Algorithm Process



Implicitization

Input: Parametric form $\mathrm{P}(u,v) = \big(a(u,v), b(u,v), c(u,v), d(u,v)\big)$ $\quad (u,v) \in [u_0, u_1] \times [v_0, v_1]$

Output: Implicit equation $\mathrm{F}(x,y,z,w) = 0$

$\boldsymbol{P(u,v)}$

$$\begin{aligned} f &\coloneqq d(u,v)x - a(u,v)w \\ g &\coloneqq d(u,v)y - b(u,v)w \\ h &\coloneqq d(u,v)z - c(u,v)w \end{aligned}$$

$D(x,y,z,w) := Dix_{u,v}(f,g,h)$

$\mathrm{F}(x,y,z,w)$

Construct moving planes

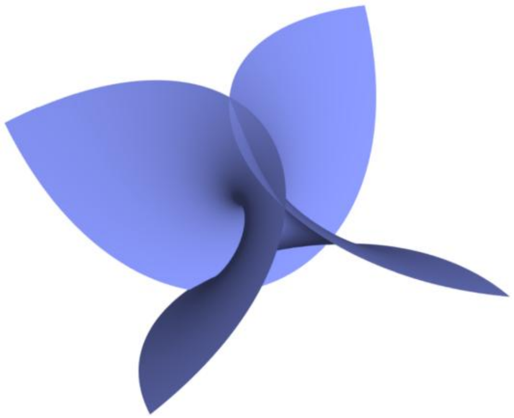
Compute Dixon matrix

Determinant

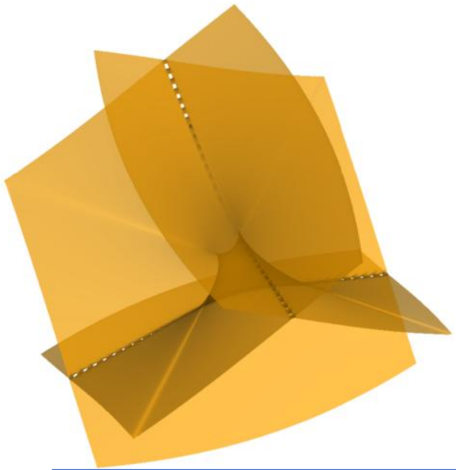
Implicitization

Input: Parametric form $P(u,v) = \big(a(u,v), b(u,v), c(u,v), d(u,v)\big)$ $\boxed{(u,v) \in [u_0, u_1] \times [v_0, v_1]}$

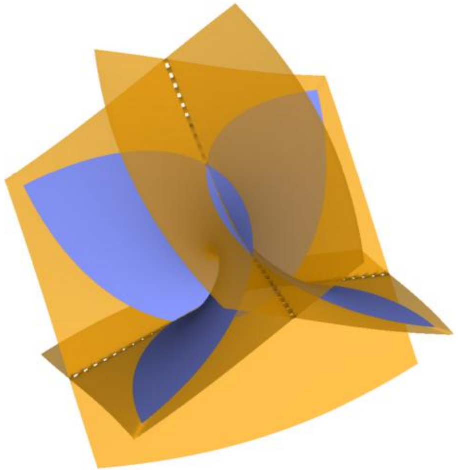
Output: Implicit equation $F(x,y,z,w) = 0$



$(u,v) \in \boxed{[0,1] \times [0,1]}$



$(u,v) \in \boxed{(-\infty,+\infty) \times (-\infty,+\infty)}$



Difference

Some parts of the implicit surface are not in the parametric surface!

Implicitization

Given a point X_0 on the implicit surface, find its parametric value (u_0, v_0).

Implicit expression

$F(X_0) = 0$

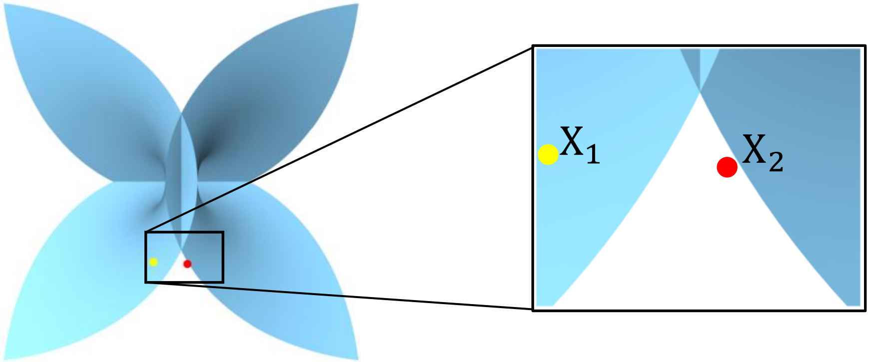
Inversion formula of Dixon matrix

$\mathbf{L}(u, v) = [1, u, v, \cdots, u^{2m-1}v^{n-1}]$

Parametric value

$X_0 = \mathrm{P}(u_0, v_0)$

- $\boldsymbol{D}(\mathrm{X}_0)\,\mathbf{L} = 0$ has a solution $(u, v) \in [u_0, u_1] \times [v_0, v_1] \Rightarrow \mathrm{X}_0$ is **on surface $\boldsymbol{P}$**.
- All real solutions of $\boldsymbol{D}(\mathrm{X}_0)\mathbf{L} = 0$ are outside $[u_0, u_1] \times [v_0, v_1] \Rightarrow \mathrm{X}_0$ is **outside surface $\boldsymbol{P}$**.



Topology Determination

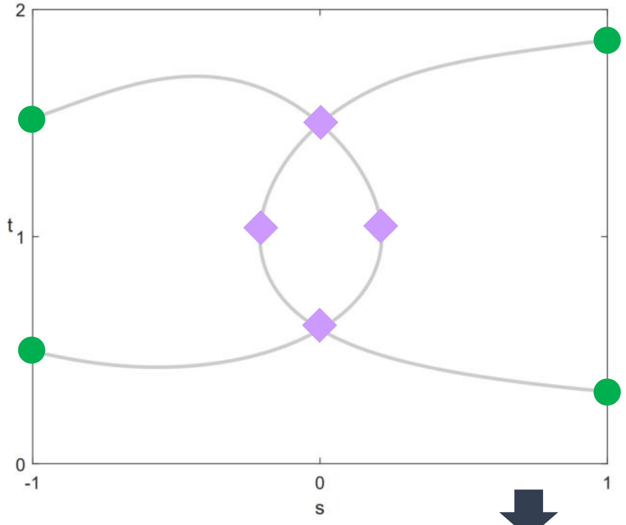
Determine the topology of $\boxed{F\big(x(s,t),y(s,t),z(s,t),w(s,t)\big)=0}$

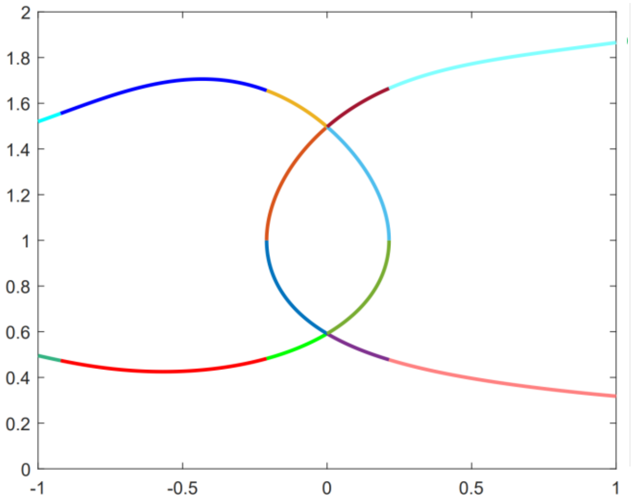
$$R(s) \coloneqq \mathrm{Res}_t\left(g(s,t),\frac{\partial g}{\partial t}(s,t)\right)$$

$R(s)=0 \Rightarrow$ ◆ Critical points $\begin{cases} \text{● Left auxiliary points} \\ \text{● Right auxiliary points} \end{cases}$

$\boxed{\begin{aligned} g(s_0,t)&=0 \\ g(s_1,t)&=0 \end{aligned}} \Rightarrow$ ● Boundary points

Acceleration: Employ parallel computation in the tracing step



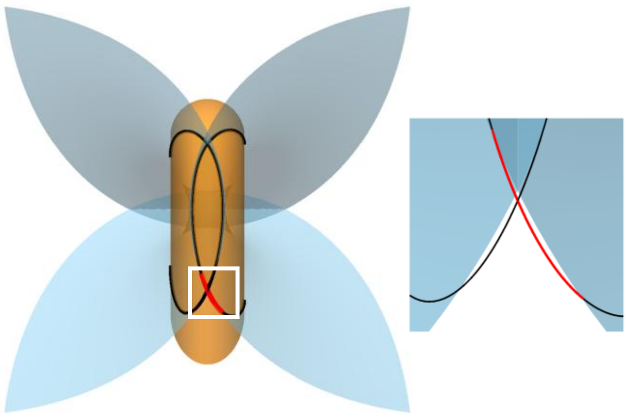


Curve Clipping

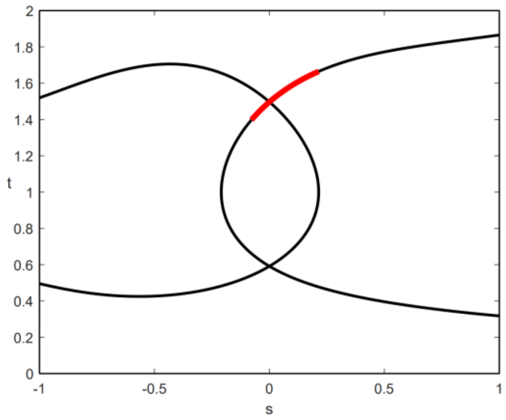
Have: $(u, v) \in (-\infty, +\infty) \times (-\infty, +\infty)$ & $(s, t) \in [s_0, s_1] \times [t_0, t_1]$

Want: $(u, v) \in [u_0, u_1] \times [v_0, v_1]$ & $(s, t) \in [s_0, s_1] \times [t_0, t_1]$

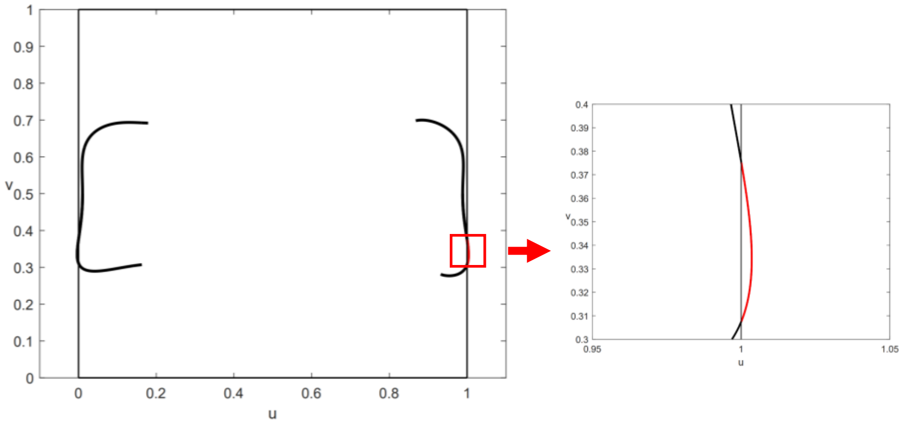
Need: **Clip the curve to fit the parametric domain** $[\boldsymbol{u_0}, \boldsymbol{u_1}] \times [\boldsymbol{v_0}, \boldsymbol{v_1}]$



Proj. to (s, t):



Proj. to (u, v):



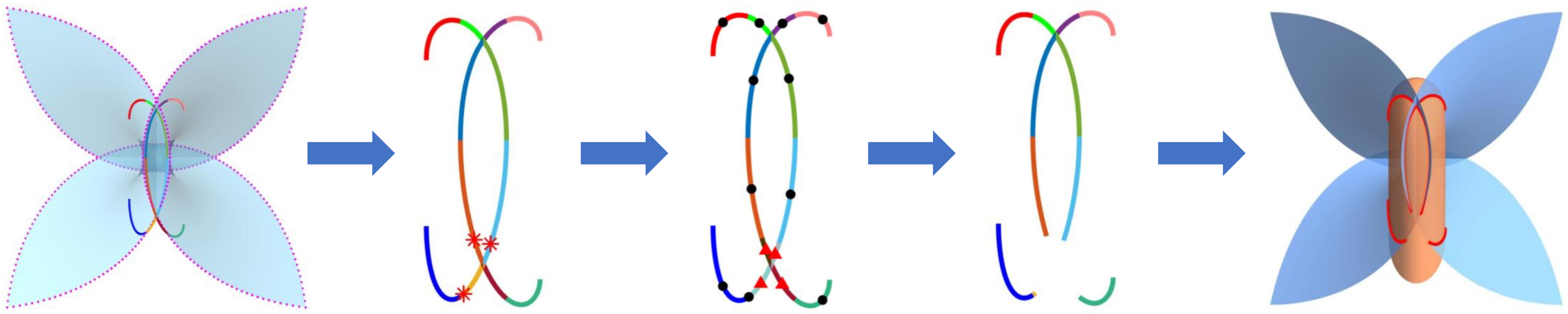
Curve Clipping

Split $\bar{C}$

- Sample (u, v) on **four boundary lines** of surface $\mathrm{P}(u, v)$
- Find **splitting points** on $\bar{C}$

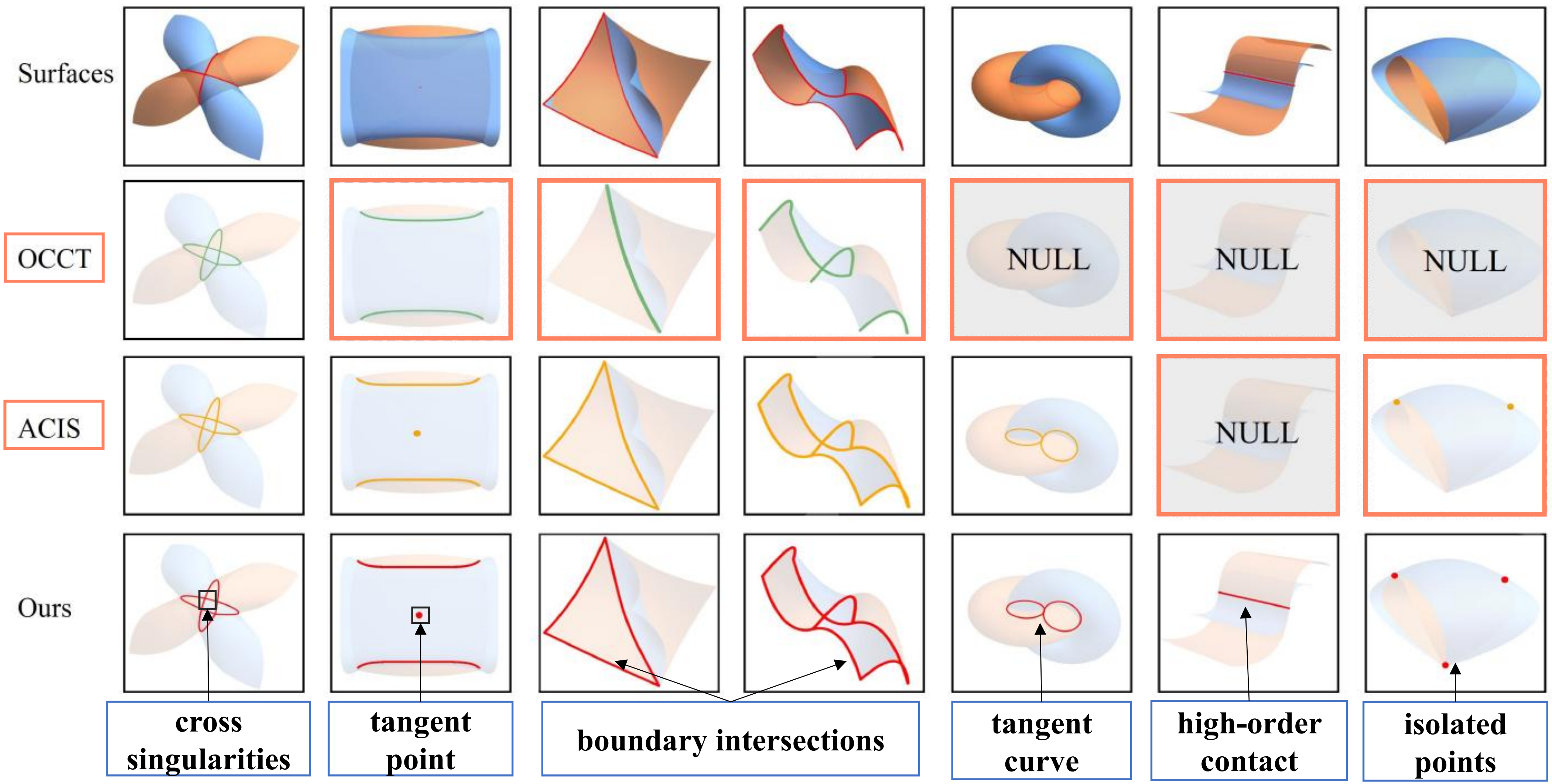
Clip $\bar{C}$

- Choose a **representative point** on each branch
- Compute the parametric value (u, v) using **Dixon matrix**



RESULTS

Topology Correctness



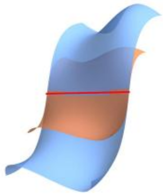
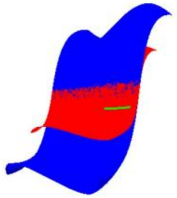
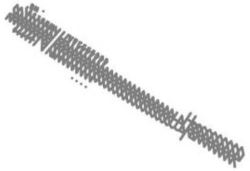
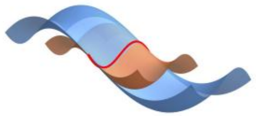
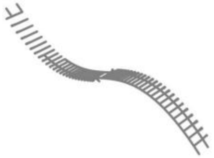
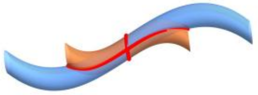
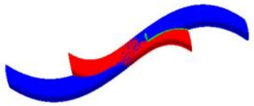
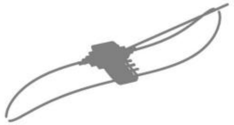
Performance Comparisons

Methods	
OCCT	Open CASCADE Technology
ACIS	3D ACIS Modeler
SISL	SINTEF Spline Library
Mesh Booleans	Interactive and Robust Mesh Boolean Algorithm
AA-GPU	Affine Arithmetic-based B-Spline Surface Intersection

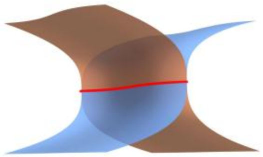
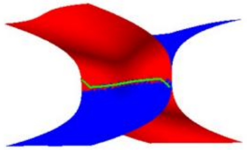
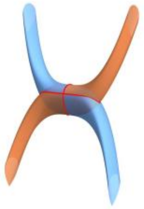
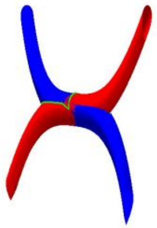
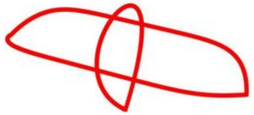
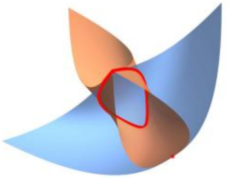
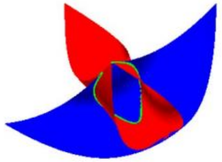
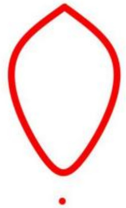
Performance Comparisons

Surfaces	AA-GPU	Mesh Booleans	OCCT	SISL	ACIS	Ours
 example (2) (2,2)	0.427	0.016	NULL 0.003	NULL 0.260	0.016	a tangential curve 0.015
 example (3) (2,2)	0.042	0.013	0.026	0.686	0.008	two intersect curves 0.027
 example (6) (2,2)	0.173	0.017	0.030	NULL N/A	0.046	four boundary curves 0.026

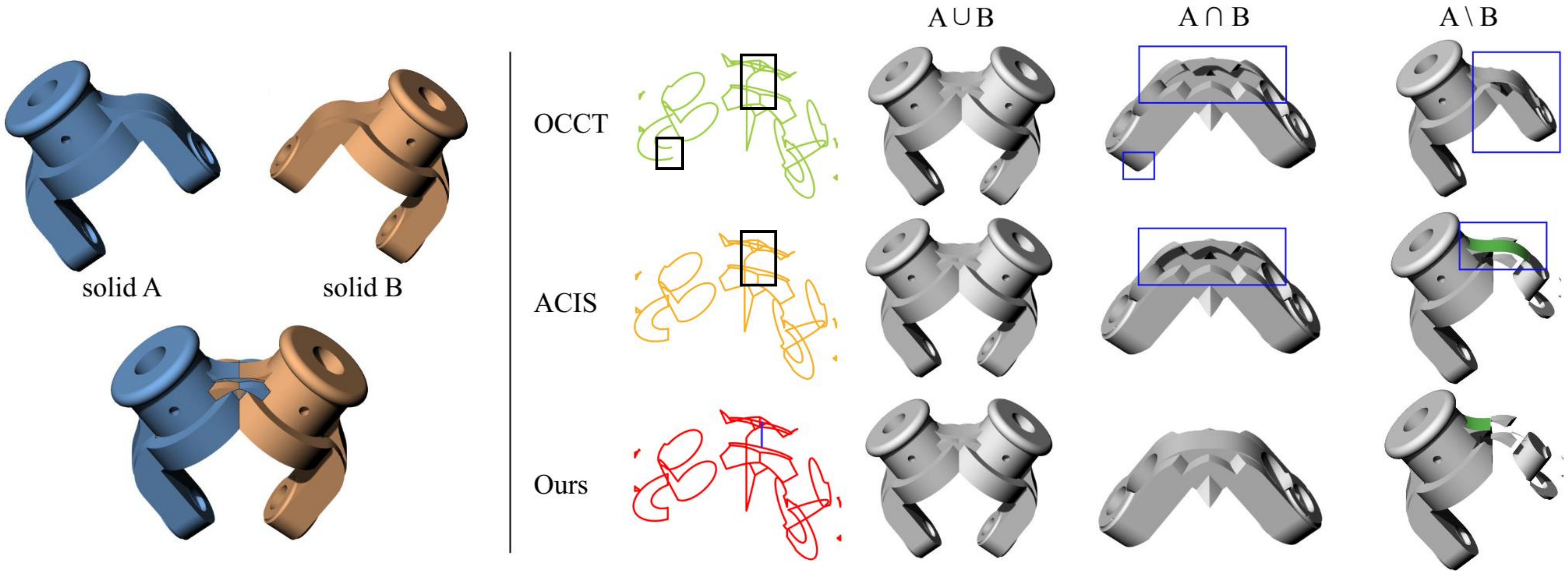
Performance Comparisons

Surfaces	AA-GPU	Mesh Booleans	OCCT	SISL	ACIS	Ours
 example 1 (3,3)	 1.451	 0.011	NULL 0.003	NULL N/A	 0.003	 a tangent line 0.036
 example 2 (3,3)	 1.775	 0.038	NULL 0.003	NULL N/A	 0.005	 a tangential curve 0.082
 example 3 (3,3)	 1.836	 0.008	 0.076	NULL 3.612	 0.074	 a loop and two curves 0.054

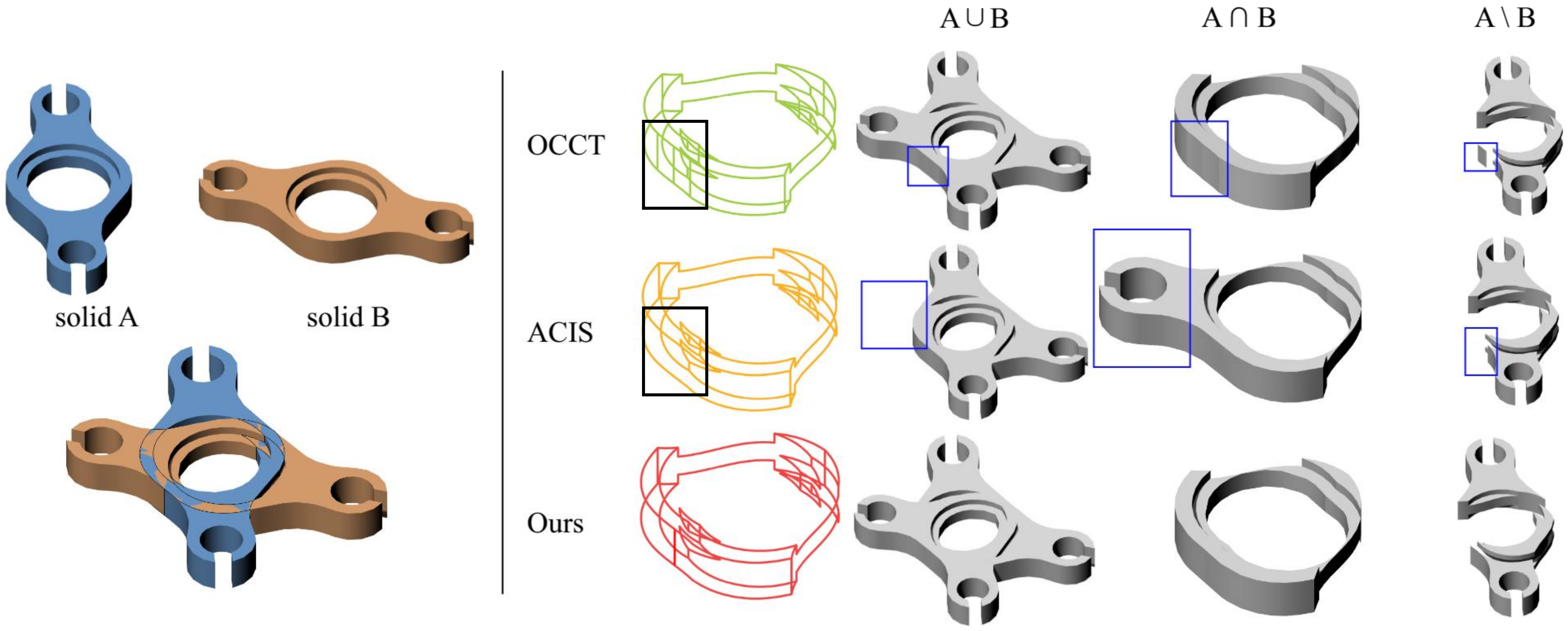
Performance Comparisons

Surfaces	AA-GPU	Mesh Booleans	OCCT	SISL	ACIS	Ours
 example 4 (3,3)	 0.388	 0.013	NULL 0.002	NULL N/A	NULL 0.003	 a tangential curve 0.019
 example 5 (3,3)	 0.532	 0.021	 0.040	NULL N/A	 0.054	 two loops 0.103
 example 6 (2,2)	 0.254	 0.012	 0.023	NULL 0.288	 0.008	 a loop and a point 0.514

Application



Application



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CONCLUSION

Conclusion

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Ours = **Algebraic** + **Marching**

Algorithm

- Implicitization with moving planes
- Topology determination of curves
- Curve clipping using Dixon matrix

singular point
tangent point
high-order contact
boundary curve
multiple branches
...

Limitation

- Computation efficiency

THANK YOU!